

INTEREST

SIMPLE INTEREST

Definitions:

Simple interest: fee charged for use of money on loans of shorter duration. It is denoted by the symbol **I**.

Principal: the money that is borrowed in a loan. It is denoted by the symbol **P**.

Interest rate: percent of the principal paid per time period. It is denoted by the symbol **r**. The symbol **t** is used for the time period.

Future value (amount): the principal and the interest of the loan. Can be denoted by the symbols **FV** or **A**.

Formulas and Examples:

$$I = Prt \quad (\text{simple interest})$$

Principal x rate x time

★ Remember that the time units for **r** and **t** must be consistent, so if you use months for one, you must use months for the other.

Example 1: Compute the interest paid on a loan of \$1400 at a 9% (per year) interest rate for 18 months.

Step 1: Write down the numbers and what they represent:

$$\begin{aligned} P &= \$1400 \\ r &= 9\% = .09 \\ t &= 18 \text{ months} = 18/12 = 1.5 \text{ years} \end{aligned}$$

Step 2: Write the equation down:

$$I = Prt$$

Step 3: Substitute and solve for the unknown:

$$\begin{aligned} I &= 1400 \times .09 \times 1.5 = 189 \\ \text{so, the amount of interest paid is } &\$189. \\ A &= P(1 + rt) \quad (\text{future value}) \end{aligned}$$

Example 2: Find the future value of a \$2400 loan for 9 months at 11% interest rate.

$$\begin{aligned} \text{Step 1: } P &= \$2400 \\ r &= 11\% = .11 \\ t &= 9 \text{ month} = 9/12 = .75 \text{ years} \end{aligned}$$

$$\text{Step 2: } A = P(1 + rt)$$

$$\begin{aligned} \text{Step 3: } A &= 2400(1 + .11 \times .75) = 2598 \\ \text{so, the future value is } &\$2,598 \end{aligned}$$

COMPOUND INTEREST

Definitions:

Compound interest: interest that is computed at specified periods of time and is added to the principal. This new total becomes the principal for the next time period and the process starts all over again. There are many ways to compound interest: annually, semiannually, quarterly, or monthly.

Interest rate per compound period: annual rate divided by the number of compound periods per year.

Present value: principal that must be invested now to accumulate the amount due, denoted by the symbol **P** or **PV**.

Formulas and Examples:

$$A = P(1 + r/m)^{mt} \text{ or } P(1 + i)^n \quad (\text{compound interest - FV})$$

- r** = annual interest rate
- m** = number of times compounded per year
- i** = interest rate per period
- n** = number of periods
- t** = number of years
- A** = amount at the end of n compound periods (FV)
- P** = principal (PV)

Example 1: \$800 is invested at 12% for two years. Find the amount at the end of 2 years compounded a) annually, b) semiannually, and c) quarterly.

Step 1:

$$\begin{aligned} r &= 12\% = .12 \\ m &= \text{a) } 1 \quad \text{b) } 2 \quad \text{c) } 4 \\ i &= \text{a) } 12\% \quad \text{b) } 6\% \quad \text{c) } 3\% \\ n &= \text{a) } 1 \quad \text{b) } 2 \quad \text{c) } 4 \\ t &= 2 \\ P &= 800 \end{aligned}$$

Step 2: $A = P(1 + r/m)^{mt}$

Step 3: Solve for A:

$$\begin{aligned} \text{a) } A &= 800(1 + .12/1)^{1(2)} = 800(1.12)^2 = \$ 1003.52 \\ \text{b) } A &= 800(1 + .12/2)^{2(2)} = 800(1.06)^4 = \$ 1009.98 \\ \text{c) } A &= 800(1 + .12/4)^{4(2)} = 800(1.03)^8 = \$ 1013.42 \end{aligned}$$

★ Use the same formula for the present value.

Example 2: How much should Josh invest at 8% compounded quarterly, so that he will have \$5000 at the end of 7 years?

Step 1:

$$\begin{aligned}r &= 8\% = .08 \\M &= 4 \\i &= 2\% \\n &= 28 \\t &= 7 \\A &= 5000\end{aligned}$$

$$\text{Step 2: } A = P (1 + r/m)^{mt}$$

$$\begin{aligned}\text{Step 3: } 5000 &= P (1 + .02)^{28} \\5000 &= 1.741 P \\P &= \$ 2871.88\end{aligned}$$