

STRAIGHT LINES

Let's look at some equations:

$$y = -3x + 5$$

$$3x - 4y = 9$$

$$y = \frac{-2}{3}x$$

These are called first degree equations because the powers of the x 's are (1). All of the above equations can be written as: $y = mx + b$, where m is the slope of the line and b is the y -intercept (where the line crosses the y -axis):

$$y = -3x + 5$$

$$\text{slope} = -3 \text{ and } y\text{-intercept} = 5$$

$$3x - 4y = 9$$

$$\text{slope} = \frac{3}{4} \text{ and } y\text{-intercept} = \frac{-9}{4}$$

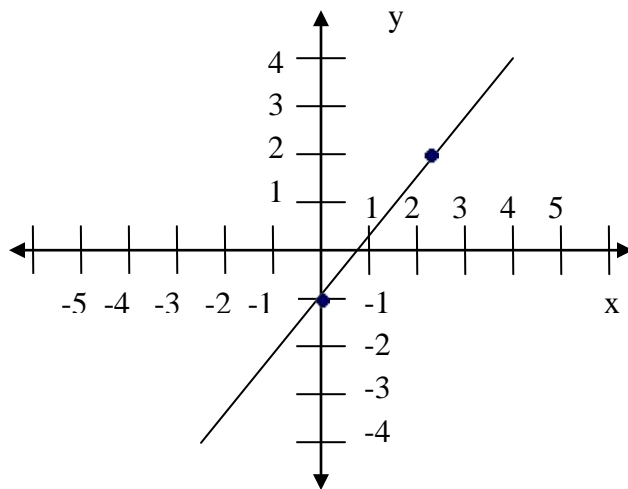
$$y = \frac{-2}{3}x$$

$$\text{slope} = \frac{-2}{3} \text{ and } y\text{-intercept} = 0$$

SLOPE:

$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{\text{vertical movement; upward (positive) or downward (negative)}}{\text{horizontal movement; forward (to the right ALWAYS)}}$$

Example 1: Graph the line whose y -intercept = -1 and slope = $3/2$



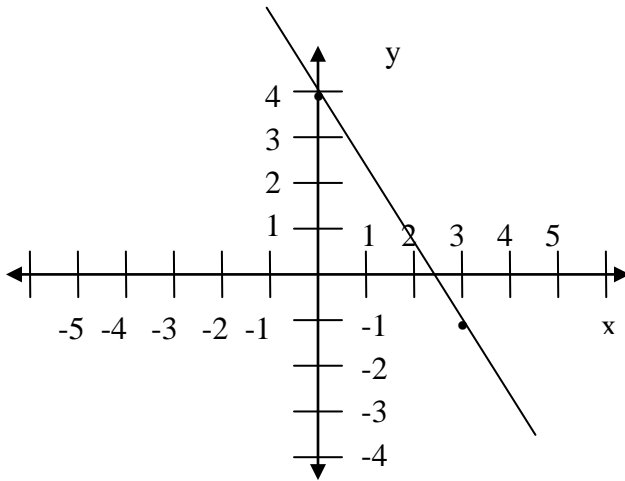
Notice that:

* y -intercept = -1

* go up 3 over 2

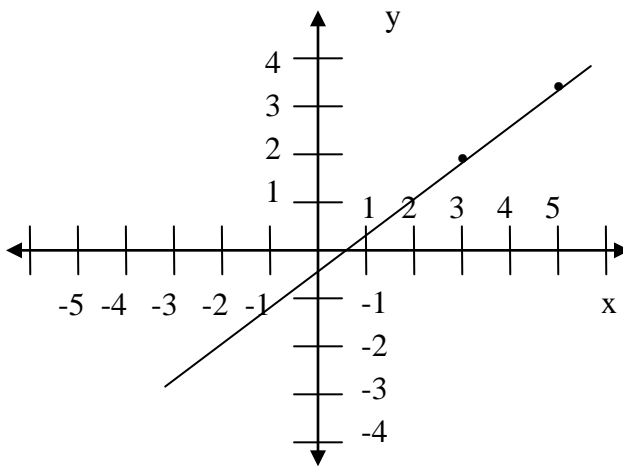
(rise = 3, run = 2)

Example 2: Graph the line whose y-intercept = 4 and slope = $-5/3$



Notice that:
*y-intercept = 4
*go down 5 over 3

Example 3: Graph the line whose slope = $1/2$ and passes through the (3, 2)



Notice that:
* point (3, 2)
* go up 1 over 2

To find the slope between two points (x_1, y_1) and (x_2, y_2) use the formula: $m = \frac{y_2 - y_1}{x_2 - x_1}$.

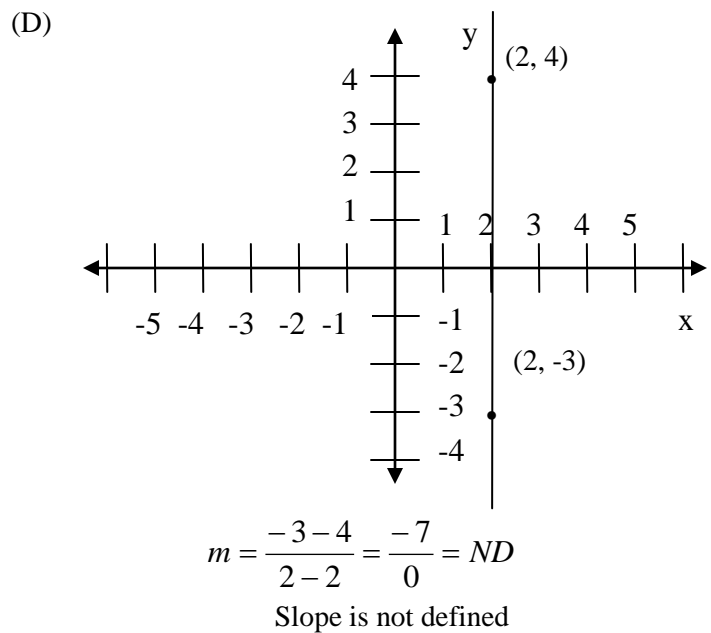
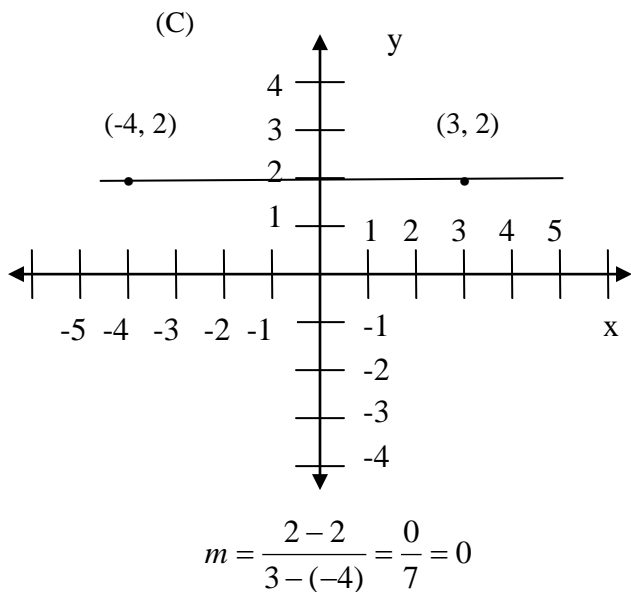
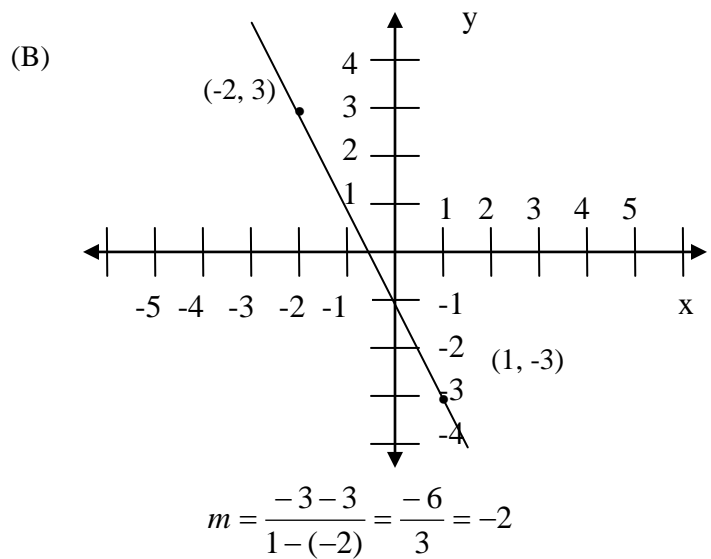
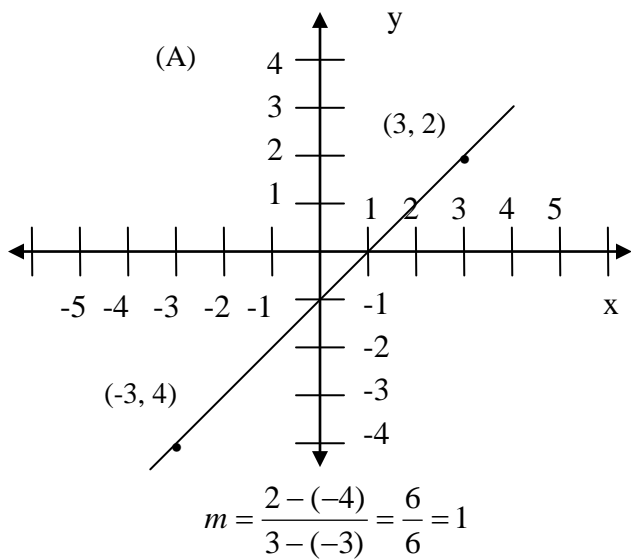
Example 4: Sketch a line through each pair of points and find the slope of each line:

(A) $(-3, -4), (3, 2)$

(B) $(-2, 3), (1, -3)$

(C) $(-4, 2), (3, 2)$

(D) $(2, 4), (2, -3)$



Notice that:

- The slope (see above) can be one of four choices: (A) positive, (B) negative, (C) zero or (D) not defined.
- A horizontal line has a slope = 0.
- A vertical line has an undefined slope.

EQUATIONS OF THE LINE:

To find the equation of the line (if you have a point and slope) use the formula

$$y - y_1 = m(x - x_1).$$

Example 5: Find the equation of the line that has slope = 3 and passes through (2, -5). Using the above formula with $m=3$, $x_1 = 2$ and $y_1 = -5$ gives:

$$\begin{aligned} y - (-5) &= 3(x-2) \\ y + 5 &= 3x - 6 \\ y &= 3x - 11 \text{ is your solution.} \end{aligned}$$

Example 6: Find the equation of the line that passes through (-2, 1) and (6, -5).

To use the formula above we need a point and a slope just like example 5 above. To find the slope of the line we use the slope formula and then pick any of the points for the equation:

$$m = \frac{-5-1}{6-(-2)} = \frac{-6}{8} = -\frac{3}{4}$$

if we pick (6, -5) as our point:

$$\begin{aligned} y - (-5) &= -3/4(x-6) \\ y + 5 &= -3/4x + 18/4 \\ y &= -3/4x + 9/2 - 5 \\ y &= -3/4x - 1/2 \end{aligned}$$

if we pick (-2, 1) as our point:

$$\begin{aligned} y - 1 &= -3/4(x - (-2)) \\ y - 1 &= -3/4(x + 2) \end{aligned}$$

Both answers are the same (it does not matter which point you choose).

Example 7: Find the equation of the line that passes through (3, -2) and has an x-intercept 4.

x-intercept 4 means the line passes through (4, 0):

$$m = \frac{0 - (-2)}{4 - 3} = \frac{2}{1} = 2$$

$y - (-2) = 2(x-3)$	<i>Point-slope form</i>	point (3, -2), slope = 2
$y + 2 = 2x - 6$		
$y = 2x - 8$	<i>Slope-intercept form</i>	slope = 2, y-intercept = -8
$y - 2x + 8 = 0$	<i>General form</i>	

PERPENDICULAR AND PARALLEL LINES:

Two lines are Parallel if and only if they have the same slope.
Two lines are Perpendicular if and only if they have negative reciprocal slopes.

Example 8: Find the equation of the line that passes through (5, 2) and:
(A) Parallel to the line passing through (4, -1) and (-8, 5).
(B) Perpendicular to the line passing through (4, -1) and (-8, 5).

$$m = \frac{5 - (-1)}{-8 - 4} = \frac{6}{-12} = -\frac{1}{2}$$

1. The two lines are parallel; they have the same slope; $m = -1/2$.

$$\begin{aligned}
 y - (-2) &= -1/2 (x-5) \\
 y + 2 &= -1/2 x + 5/2 \\
 y &= -1/2 x + 1/2
 \end{aligned}$$

2. The two lines are perpendicular; they have negative reciprocal slopes;
 $m = +2/1 = 2$.

$$\begin{aligned}
 y - (-2) &= 2 (x - 5) \\
 y + 2 &= 2x - 10 \\
 y &= 2x - 12
 \end{aligned}$$

Example 9: Find the equation of the line that passes through (-3, 5) and perpendicular to the line L: $3x - 2y = 5$.

First find the slope of L by writing $3x - 2y = 5$ in the equivalent slope-intercept form $y = m x + b$:

$$\begin{aligned}
 3x - 2y &= 5 \\
 -2y &= -3x + 5 \\
 y &= 3/2 x - 5/2 \quad m = 3/2
 \end{aligned}$$

Our line is perpendicular to L and has a slope of $-2/3$:

$$\begin{aligned}
 y - 5 &= -2/3 (x + 3) \\
 y - 5 &= -2/3 x - 2 \\
 y &= -2/3 x + 3
 \end{aligned}$$

Example 10: Find the equation of the line that is the perpendicular bisector of (-3, 2) and (7, -4).

$$m = \frac{-4 - 2}{7 - (-3)} = \frac{-6}{10} = -\frac{3}{5}$$

Our line has slope of $5/3$ (perpendicular) and passes through the midpoint which is:

$$\left(\frac{-3 + 7}{2}, \frac{2 - 4}{2} \right) = \left(\frac{4}{2}, \frac{-2}{2} \right) = (2, -1)$$

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

is the midpoint formula

$$\begin{aligned}
 y + 1 &= 5/3 (x - 2) \\
 y + 1 &= 5/3 x - 10/3 \\
 y &= 5/3 x - 13/3
 \end{aligned}$$