COMPLETING THE SQUARE

Example 1: $x^2 + 5x + 1 = 0$

1. Make sure the coefficient of x^2 (the number in front of x^2) is 1 (it is).
2. Take the constant (the number with no x's) to the other side.

   $x^2 + 5x + 1 - 1 = 0 - 1$

   $x^2 + 5x = -1$

3. Divide the coefficient of x by 2 and square the result: $(5/2)^2$, add to both sides:

   $x^2 + 5x + (5/2)^2 = -1 + (5/2)^2$

4. Now the left side of the equation will factor as a perfect square.

   $(x + 5/2)(x + 5/2) = -1 + 25/4$

   $(x + 5/2)^2 = -4/4 + 25/4$

   Solve for x:

$$(x+5/2)^2 = 21/4$$

$$\sqrt{(x+5/2)^2} = \pm\sqrt{21/4}$$ Take the square root of both sides and simplify.

$$x + 5/2 = \pm\sqrt{21}/\sqrt{4}$$

$$x = -5/2 \pm \sqrt{21}/2$$

$$x = \frac{-5 \pm \sqrt{21}}{2}$$

Example 2: $x^2 - 4x + 1 = 0$

$x^2 - 4x = -1$ Subtract 1 from both sides

$x^2 - 4x + 4 = -1 + 4$ Add $(4/2)^2$ to both sides (16/4 = 2)

$(x - 2)(x - 2) = 3$ Factor

$(x - 2)^2 = 3$

$\sqrt{(x-2)^2} = \pm\sqrt{3}$ Square root both sides to cancel the square

$x - 2 = \pm\sqrt{3}$

$x = 2 \pm \sqrt{3}$ Add 2 to both sides

$x = 2 + \sqrt{3}$; $x = 2 - \sqrt{3}$ Answers

Example 3: $2x^2 + 7x - 30 = 0$

$2x^2 + 7x = 30$

$x^2 + (7/2)x = 15$ Divide by 2 (see 1 above)

$x^2 + (7/2)x + (7/4)^2 = 15 + (7/4)^2$

$(x + (7/4))^2 = (289/16)$

$\sqrt{(x + 7/4)^2} = \pm\sqrt{289/16}$

$x + 7/4 = \pm 17/4$

$x = -7/4 \pm 17/4$

$x = -7/4 + 17/4$, $x = -7/4 - 17/4$

$x = 10/4$ $x = -24/4$

$\underline{x = 5/2}$ $\underline{x = -6}$