Math Explorer

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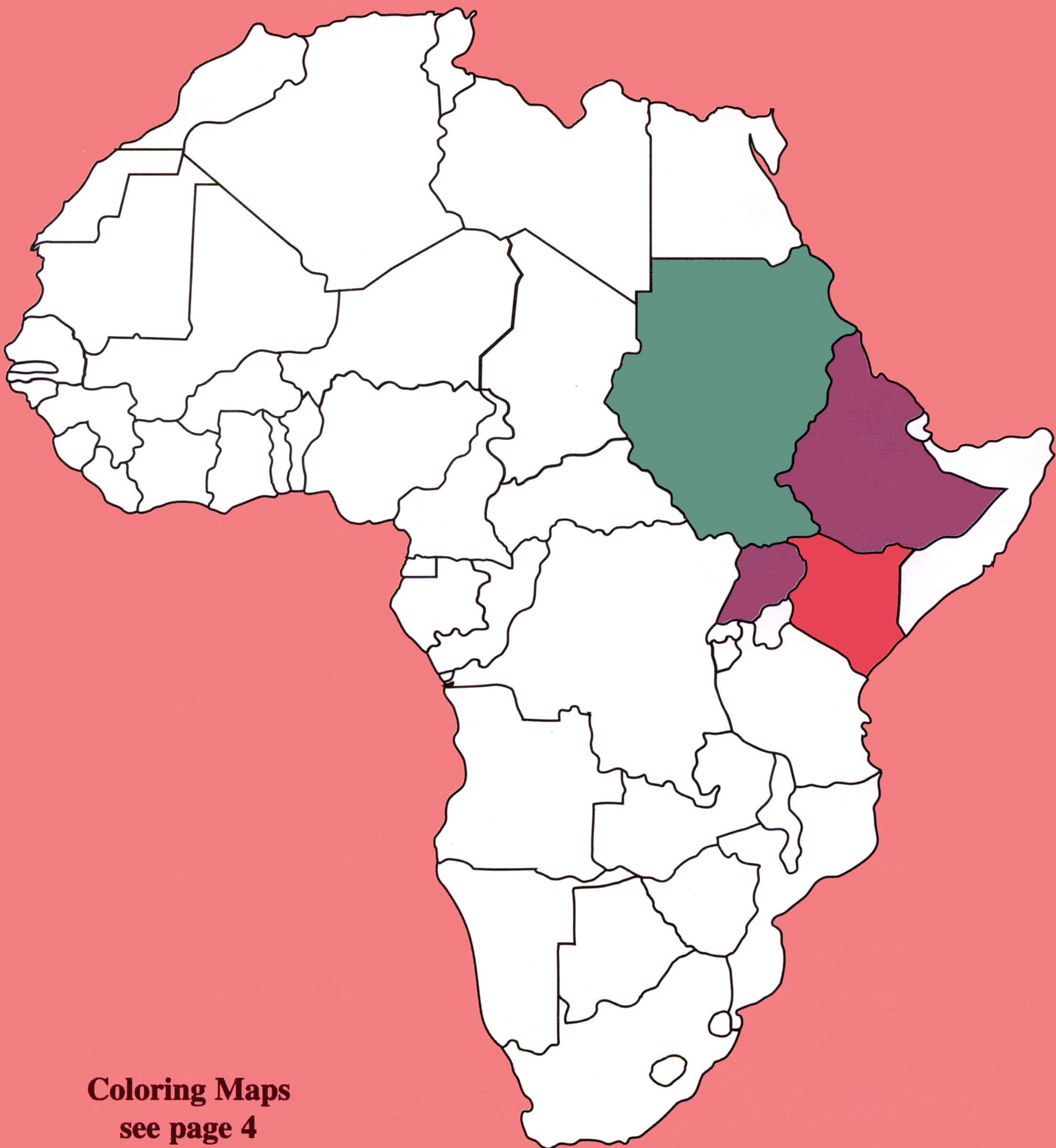

Coloring Maps
see page 4

Math Explorer

Contents

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Pierre de Fermat

by Hiroko Warshauer

Hiroko Warshauer teaches mathematics at Southwest Texas State University. She enjoys music and the arts, as well as working with students on math puzzles, problems, and activities.

Pierre de Fermat was an extraordinary mathematician who lived in France during the seventeenth century, at the same time as two other very famous mathematicians, Descartes and Pascal. Fermat made contributions in many fields of mathematics, including number theory, analytical geometry, and probability theory.

Fermat was born in 1601, in Beaumont de Lomagne, France. He was a learned man, who studied many languages, literature, and law. Although Fermat was not a professional mathematician, but a lawyer, his real passion was mathematics. He often wrote to other mathematicians about his work, but did not publish his work extensively.

Fermat did not take much time to write up his discoveries, so his explanations are often brief or incomplete. Such was the case of the problem that has been referred to as "Fermat's Last Theorem" for over three hundred years. The statement says that if the exponent n is larger than 2, then there are no rational numbers x, y, and z such that $x^n + y^n = z^n$. Notice that if n is equal to 2, then we can find rational numbers such as x=3, y=4, and z=5 such that, $3^2 + 4^2 = 5^2$. Can you find other values that will work too?

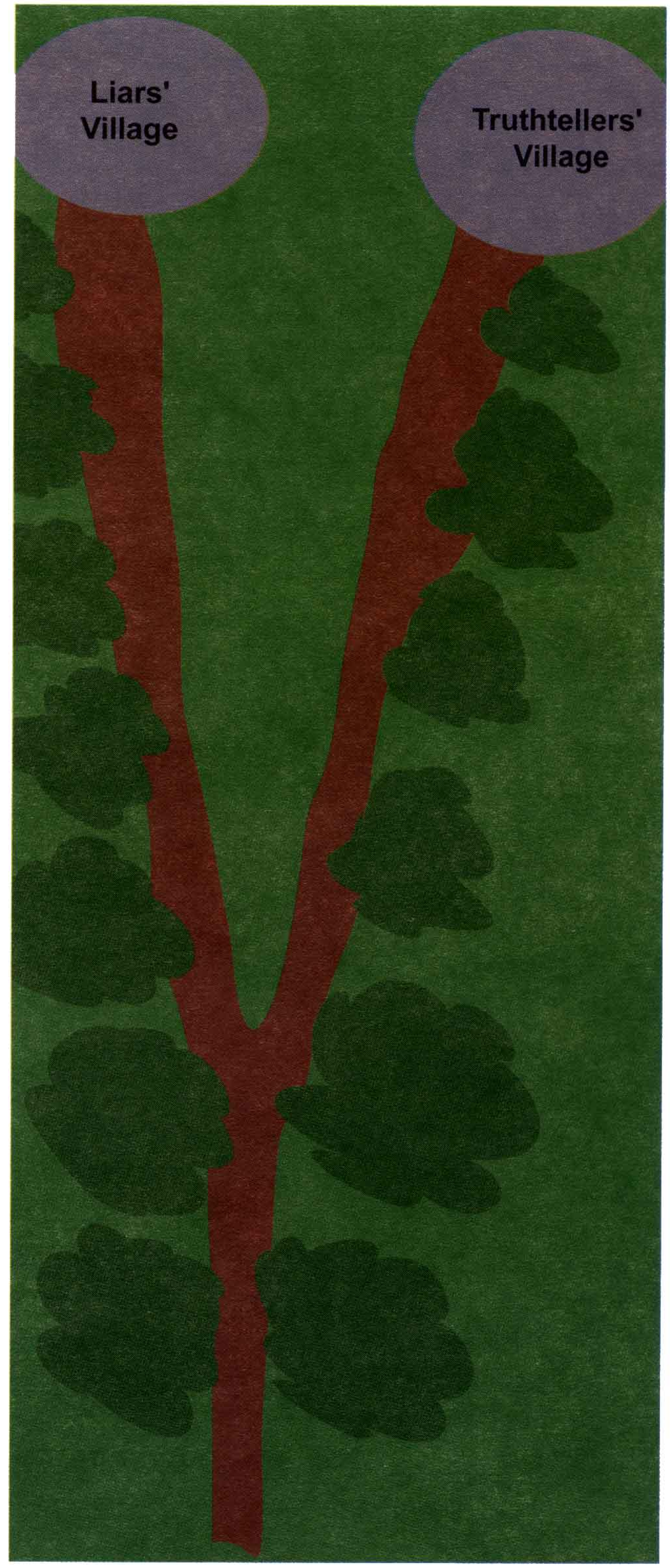
Fermat wrote that he had found a marvelous proof of this, but that he did not have enough room in the book's margin to explain why the statement above was true. Since discovering this claim by Fermat, the mathematical community spent nearly four hundred years trying to show Fermat was either wrong or right. Only this decade has a proof of Fermat's Last Theorem been discovered by the mathematician Andrew Wiles.

This might serve as a reminder that showing our work may help others understand more clearly what we are thinking!

[B] Bell, E. T., *Men of Mathematics*, Simon and Schuster, NY, NY, 1965.
[Ho] Hollingdale, Stuart, *Makers of Mathematics*, Penguin Books, NY, NY, 1989.

PROBLEMS OF THE MONTH

Send your solutions to *Math Explorer*! We will publish the best solutions each month and send a free *Math Explorer* pen to you and your math teacher if we print your solution.



1. Suppose you have a group of 4 girls and 3 boys. How many ways can you pick two people if one of them must be a girl?

2. For a given amount of money, what percent more goods can you purchase if the prices drop by 20%?

3. In fox-school, little foxes can get credit for taking three classes. There are 3 foxes taking both hen-counting and goose-catching. There are 2 taking both hole-digging and goose-catching. There are 4 taking both hen-counting and hole-digging. There are 12 taking hen-counting, 10 taking hole-digging and 7 taking goose-catching. How many little foxes go to this school, if every one of them takes at least one of these 3 subjects?

4. A farmer wants to enclose a rectangular garden with 48 feet of fencing. She plans to use a barn as one side and fencing on the other three sides. What are some possible models she can use? How long should the sides of the fence be in order get the most area possible in her garden?

5. How many ways can you draw two face cards from a well-shuffled deck of 52 cards?

6. Suppose you roll two six-sided dice and add the number of dots on the up-turned sides. What are the possible outcomes, and which sum occurs most often? Work this same problem for rolling three six-sided dice.

7. Ten suitcases arrived in a store with their 10 keys in an envelope. Every key opens just one suitcase. What is the fewest number of trials guaranteed to open every suitcase? Hint: Try this problem with 2 suitcases; 3 suitcases; etc. Do you see a pattern?

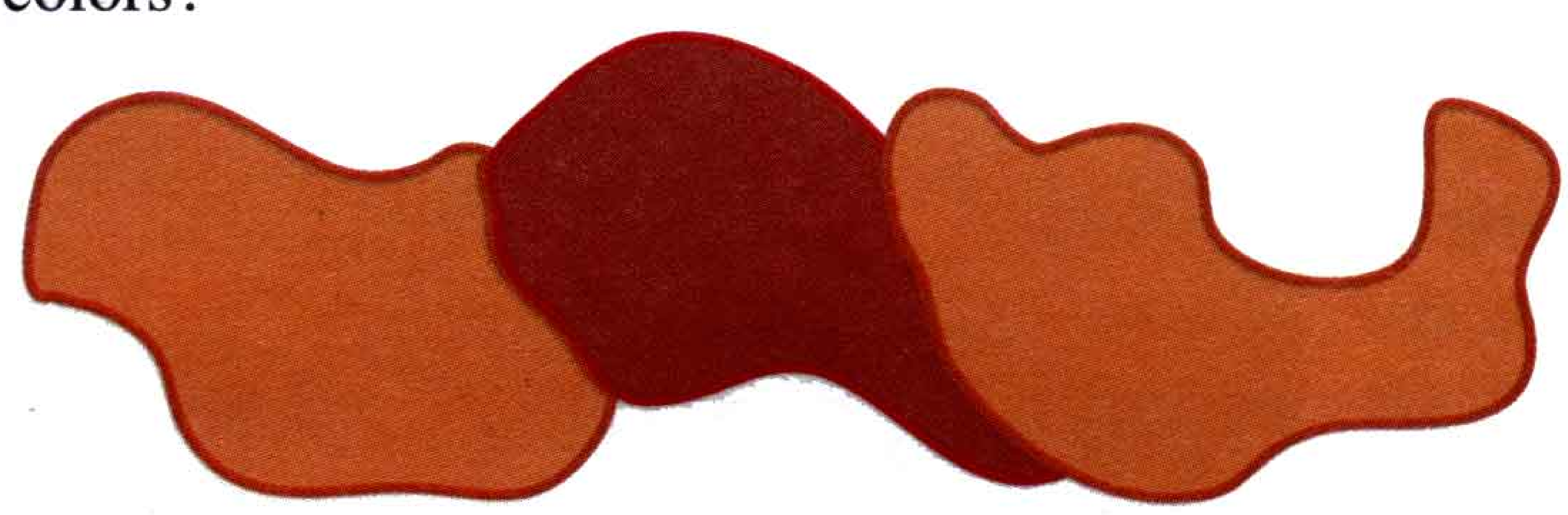
8. Ingenuity There are two villages. Everyone from the truthtellers' village always tells the truth, while everyone from the liars' village always tells lies. A man is walking along a path where the road forks. He spots a villager (but he doesn't know if the person is a truthteller or a liar), and needs to find out which path to take to get to the truthtellers' village. What question should he ask?

Coloring Maps

by Terry McCabe

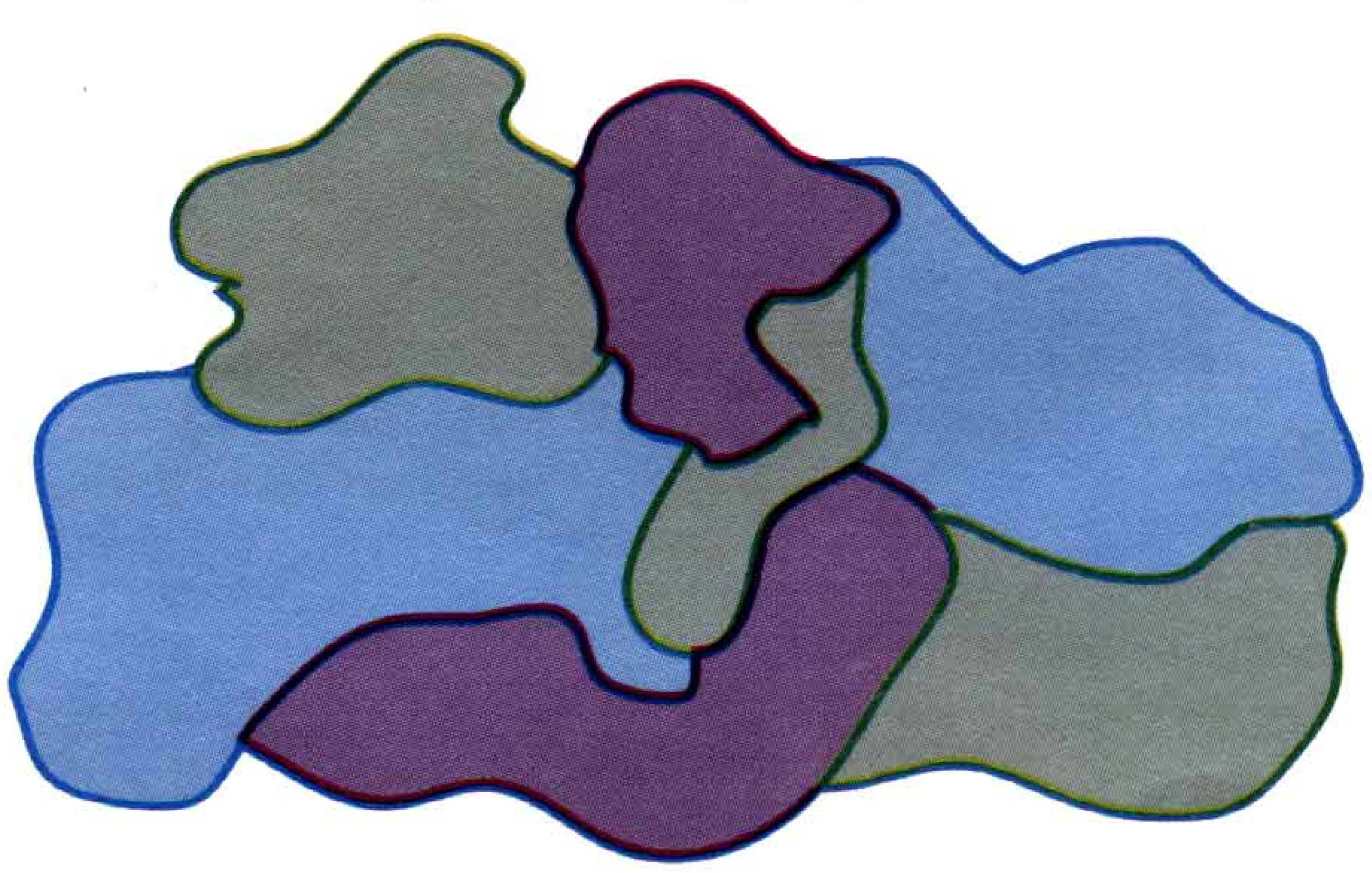
Coloring maps can be fun! You might think that the bigger your box of crayons, the better prepared you are to color a map. But suppose you want to color a map using the fewest colors possible so that no two countries with a common border are colored the same. We also assume that each country has at least one border with another country and there are no islands of countries. What is the smallest number of colors guaranteed to color such a map? This is a famous old problem.

Let's start by drawing a map with three countries (below) that can be colored with just 2 colors. Take another piece of paper and try drawing maps with 4 and 5 countries that require only 2 colors. Do you see a pattern in what a map must be like to require only 2 colors?



Your countries can be rectangular, but if they are, the map may not look realistic.

Now let's draw some maps that require exactly three colors. Draw such maps with 3, 4, 5, 6, 7 and 8 countries respectively. Is there some common feature that each of the maps in this group must have?



For many years, mathematicians believed that any map could be colored using only 4 colors. Proving this was called the "Four Color Problem". Draw some maps that require 4 colors. What is the fewest number of countries a map requiring 4 colors can have?

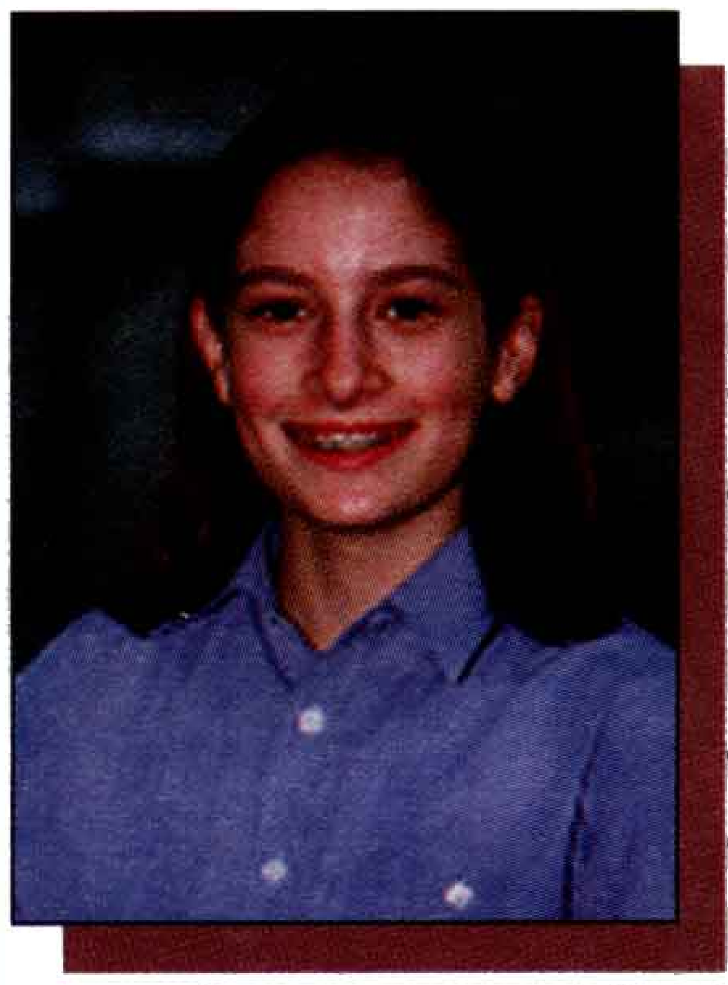
The answer to the last question is 4 countries. Did you draw such a map? If not, try to draw one now.

Draw your favorite "Hard to color Map" that requires at least 4 colors to color here! How many countries does it have?

In order to see why it was believed that 4 colors was enough, try drawing some maps that require 5 colors. It is a very interesting activity. But please do not spend more than a week or more than a pack of paper on this task.

A group of mathematicians showed in the 1980's that 4 colors is always enough! Using computers to check thousands of possibilities, they showed that no map, no matter how complicated, requires more than four colors. This solved the famous "Four Color Problem" which had baffled mapmakers for centuries.

Math Notes



Hello, my name is Jenny Mancino, and I am eleven years old (pictured at the left). This year I enrolled in the sixth grade at Grace Church School in Manhattan. Grace is from Junior-Kindergarten till the Eighth Grade. I have gone here since I was 4 years old. I was given these Math Explorer questions by my sixth grade science teacher, Mr. Diveki.

The Donut Problem

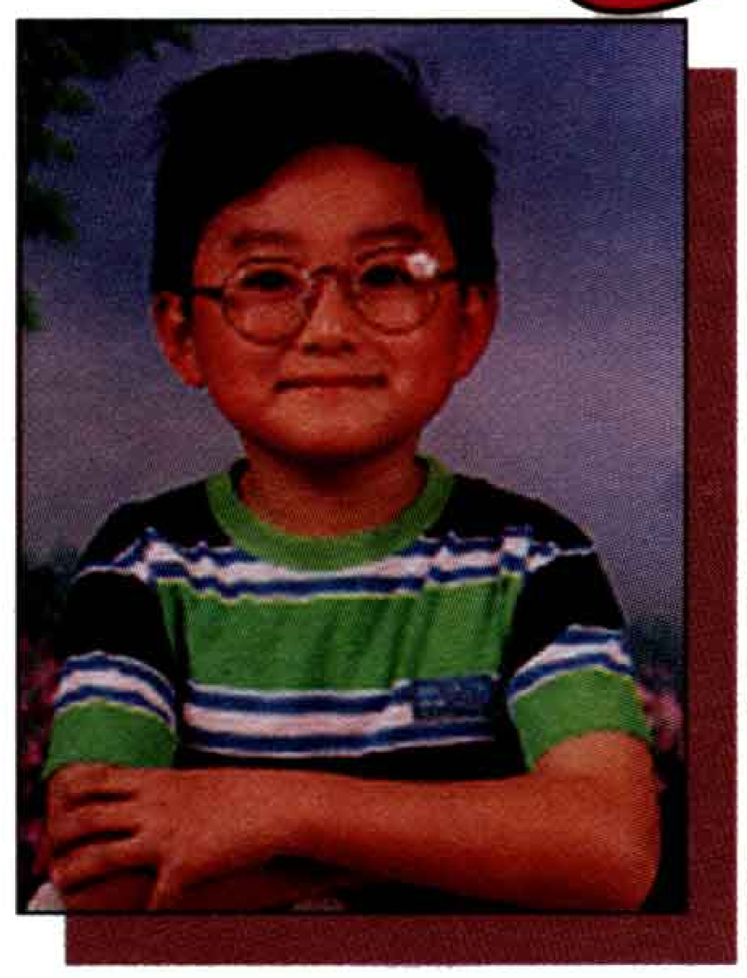
How many ways are there to select eight donuts from a collection of glazed and chocolate donuts? To select eight donuts from a collection of glazed, chocolate, and cream-filled donuts?

I made a diagram of eight donuts. I then calculated how many ways I could choose two kinds of donuts from eight. I was able to figure this out because if you have a certain number of one kind, you are able to figure out how many of the other kind(s) you have.

a) For A I figured out 9 ways to pick two kinds of donuts from eight because I could pick 0, 1, 2, ... , 8 of one kind

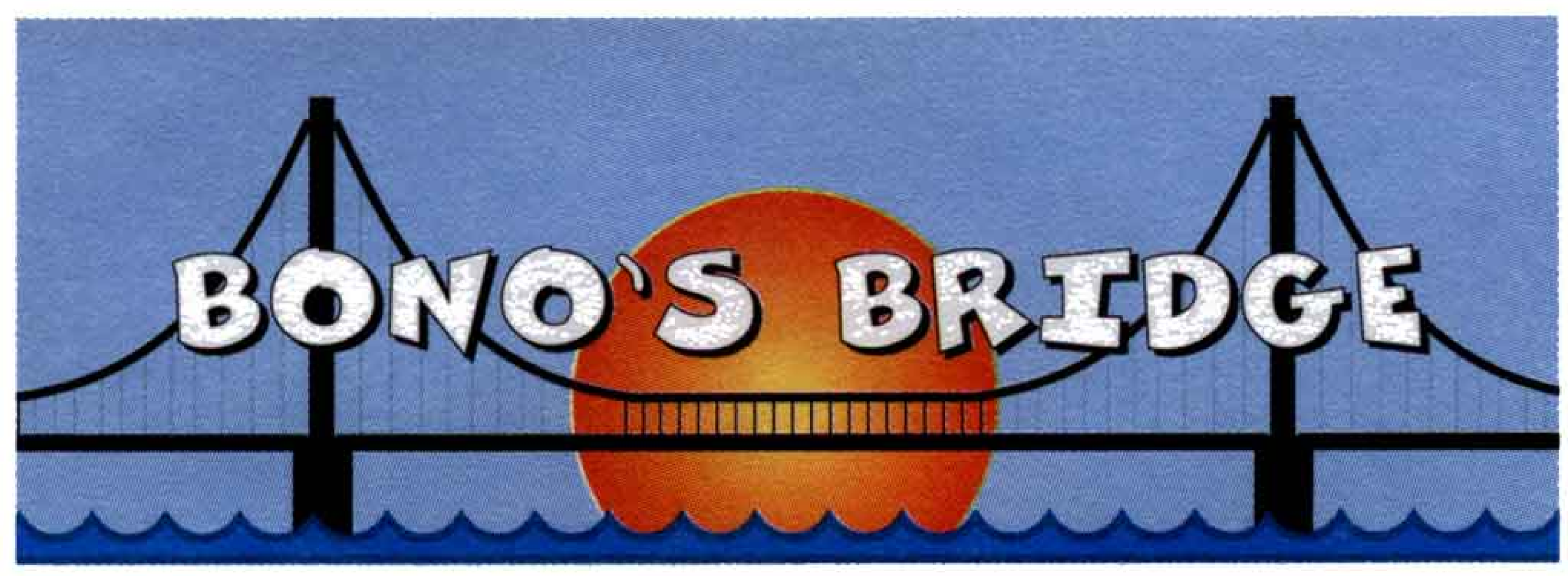
b) For B I figured out there were 45 ways to pick three kinds of donuts from eight because if I pick 0, 1, ..., 8 for kind "A", then I can pick the remaining pieces in 9, 8, ..., 1 ways. So the total is 9+8+7+6+5+4+3+2+1.

Editor's Note: Jenny submitted solutions to all 9 problems!! **Great Job, Jenny!**



Dear Dr. Warshauer,

My name is Lyndon Zhang. I am 8 years old and a student in Ms. Farrier's class at Brentwood Christian School. Here is my solution to one of the puzzles in Math Explorer.



Bono, The Edge, Adam, and Larry, the band members of "U2," must cross a bridge to get to their concert on time. Bono takes 1 minute to cross, The Edge 2 minutes, Adam 5 minutes, and Larry 10 minutes. The bridge can hold at most two people at a time. There is only one flashlight which must be used to cross the dark bridge, and when two people share the flashlight, they go at the rate of the slower person. How can all four band members cross the bridge in 17 minutes?

1. Bono and The Edge go across the bridge in 2 minutes.
2. Bono goes back in 1 minute with a flashlight.
3. Adam and Larry go across the bridge in 10 minutes.
4. The Edge goes back in 2 minutes.
5. Bono and The Edge go across the bridge in 2 minutes.

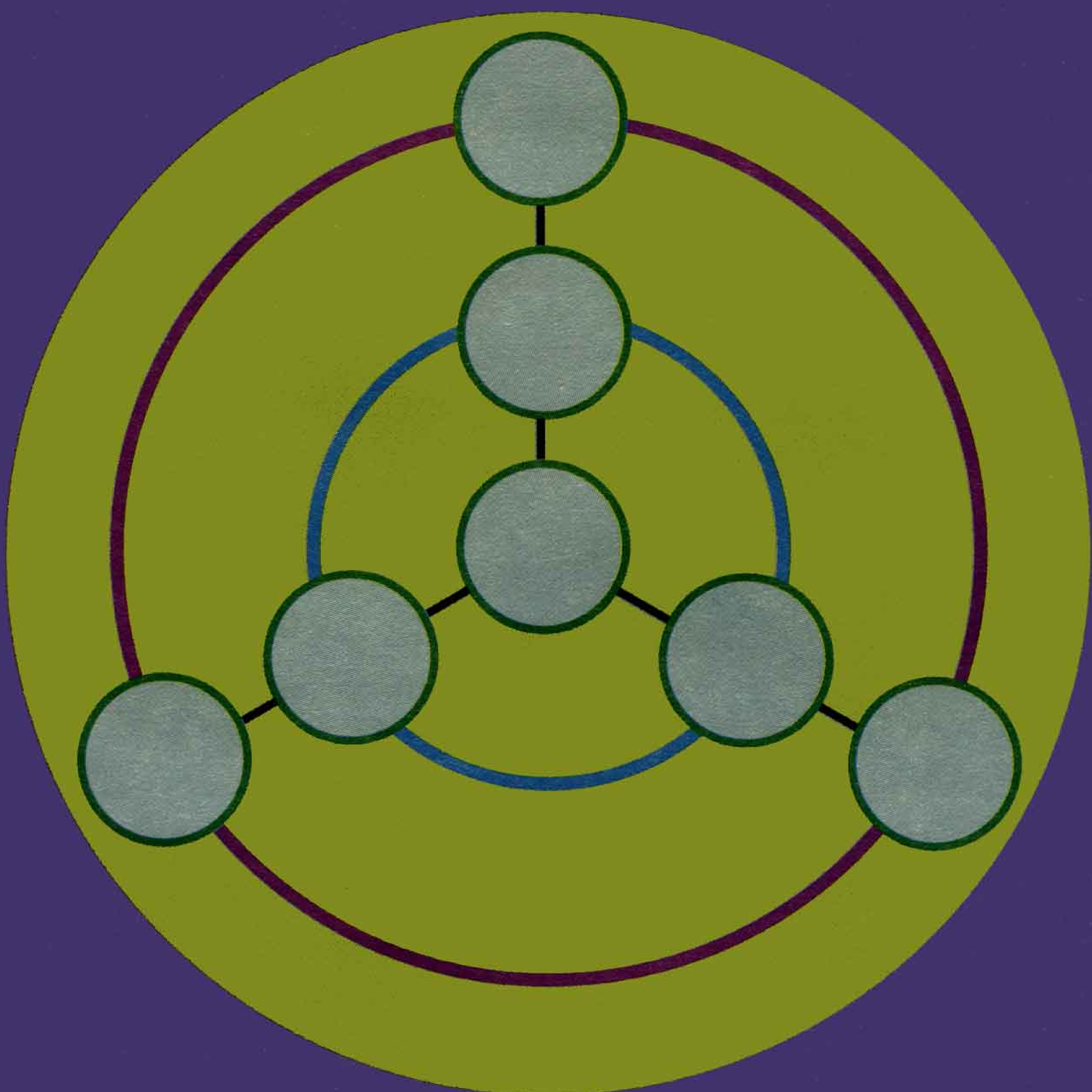
So the total is 17 minutes.

Editor's Note: Nice Work, Lyndon!

Puzzle Page

Math Explorers,

We want to print your work! Send us your own math games, puzzles, problems, and activities. If we print them, we'll send you and your math teacher free *Math Explorer* pens.



Using 1, 2, 3, 4, 5, 6, and 7, place a number in each green circle so that the sum of the numbers on each straight line of circles and on the purple and blue rings is 12.

FOUR 5'S

Use the operations

$+ \quad - \quad \times \quad \div$

and parentheses to combine the four 5's and make each equation below true. For example, we can use four 5's to make 0 like this:

$(5 - 5) \times (5 + 5) = 0$

5 5 5 5 = 2

5 5 5 5 = 3

5 5 5 5 = 4

5 5 5 5 = 5

5 5 5 5 = 6

5 5 5 5 = 7

5 5 5 5 = 9

5 5 5 5 = 10

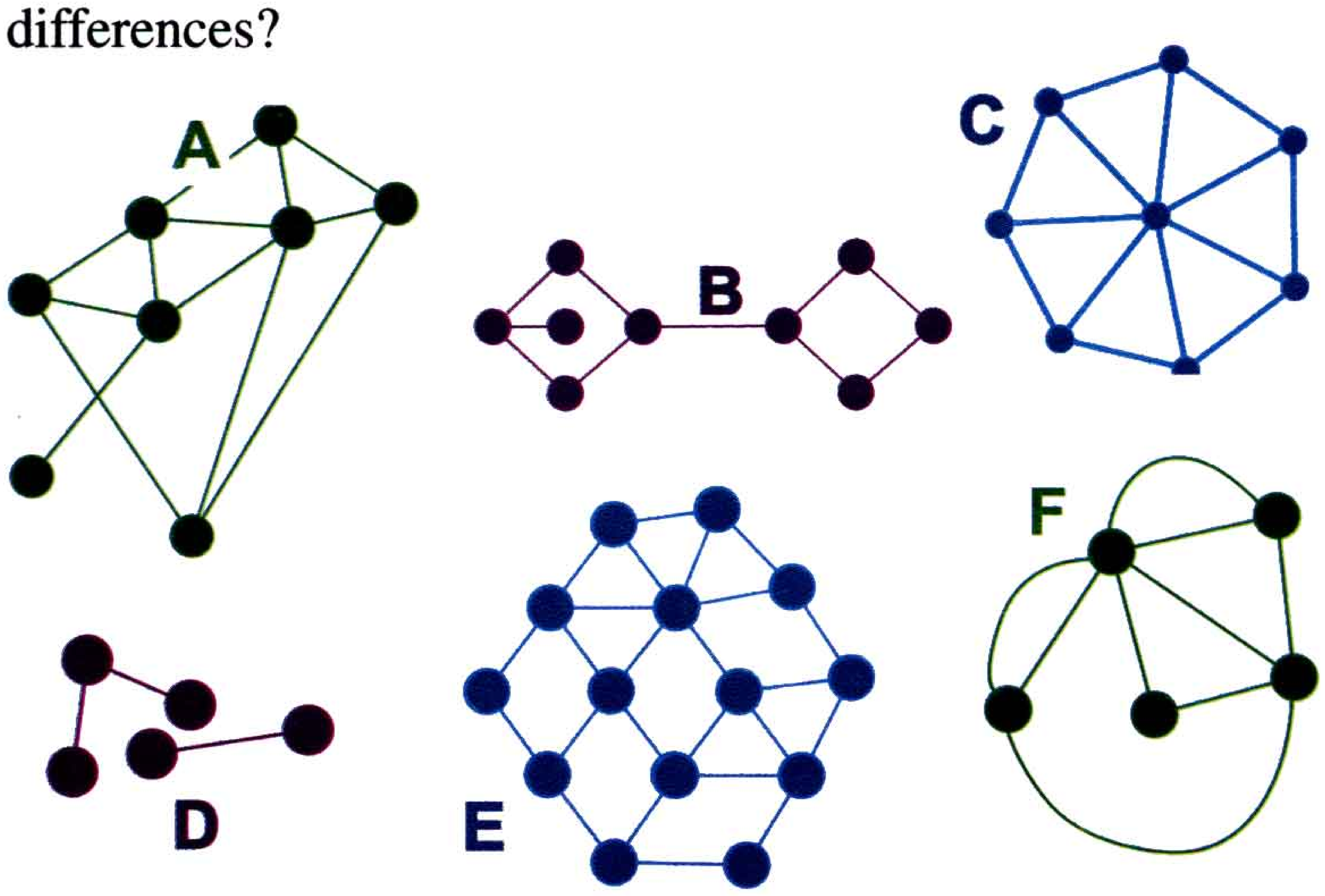
5 5 5 5 = 11

Edges and Vertices

by Eugene Curtin and Terry McCabe

Eugene Curtin and Terry McCabe teach mathematics at Southwest Texas State University. Eugene, also the Texas state chess champion, enjoys problems in graph theory and combinatorics. Terry enjoys working with students on math problems, as well as gardening and fishing.

Sometimes students ask, "What's new in math?" Most math that students learn, even in college, has been known for hundreds, if not thousands, of years. But there are many questions and problems that are relatively new, having been studied only in this century. One such group of problems can be found in graph theory which is the study of points (vertices) and lines (edges) connecting these points. Consider the graphs below: We have labeled these graphs A, B, C, D, E, and F. Do you notice anything in common among the graphs? What are differences?

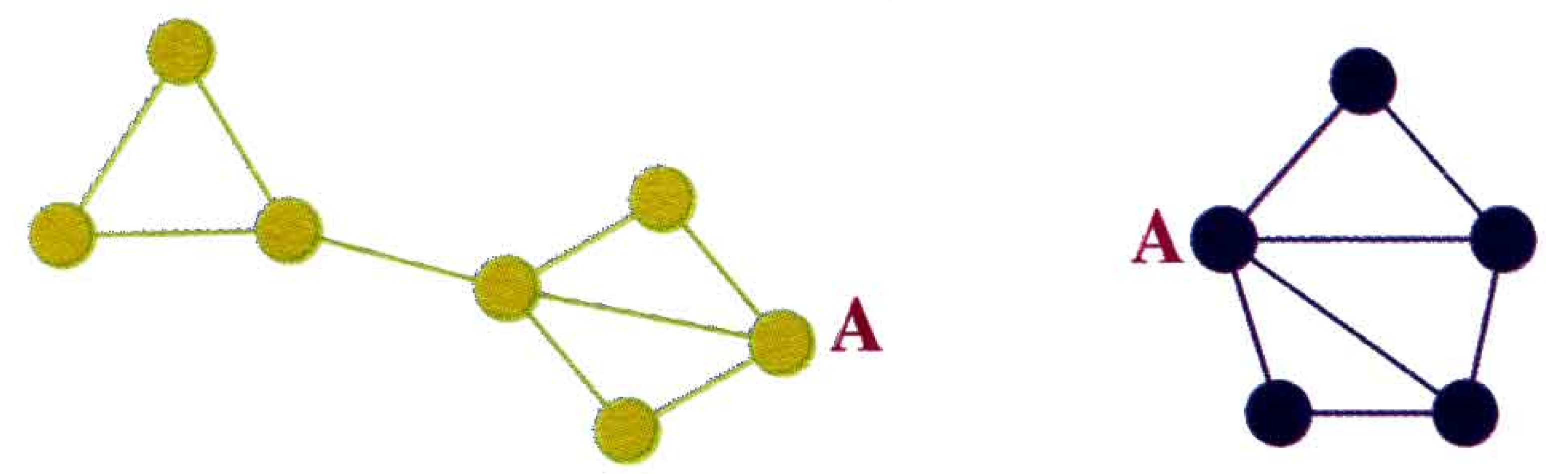


Notice that edges are allowed to cross over each other, and we also allow more than one edge to join two vertices. What do you think we would mean if we said a graph is connected? Which of the graphs would you call connected?

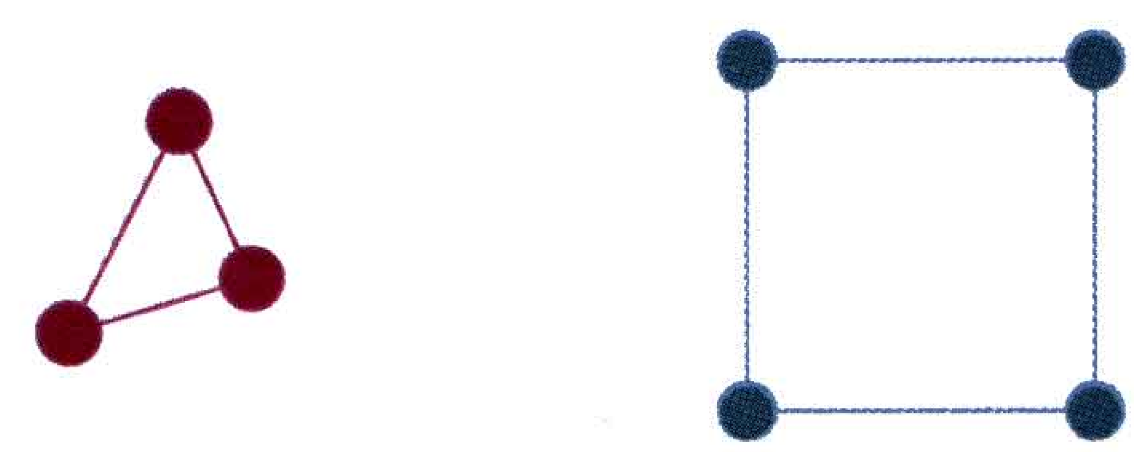
Draw at least 10 different graphs, each containing only 4 vertices. You may need to use other paper. What does it mean to say the graphs are different?

Consider a vertex (call it A) of a graph and count the number of edges connected to A. This number is called the **valence** of the vertex A. Calculate the valence of the vertices on the blue graphs above. What do you notice about these valences? Count the number of vertices in each graph that have an odd valence. Do you see any patterns?

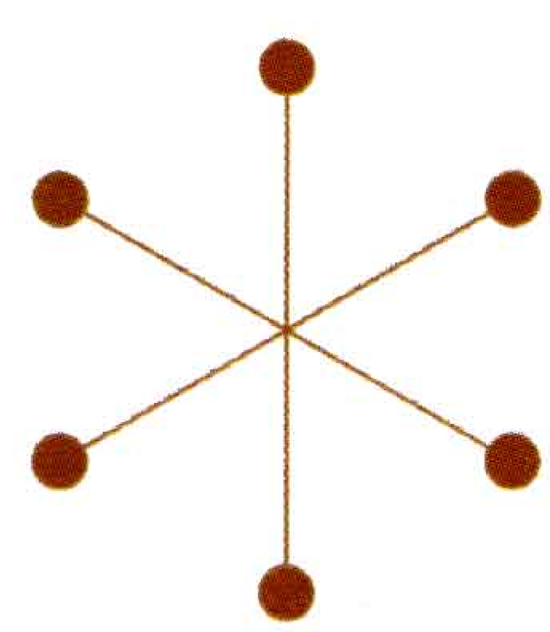
We will now talk only about graphs that are connected, that is, graphs in which you can get from one vertex to any other by moving along edges. The distance from a vertex A to a vertex B is the smallest number of edges it can take to get from A to B. For each graph below, determine the valence of each vertex and the distance from A to each of the other vertices.



A very interesting question is: How many vertices can a graph have if the valence of each vertex must be no greater than 2 and the distance between any two vertices is no greater than 2? The graphs below have these properties. Draw graphs of this type with more vertices. How many vertices can you get?



Another question to ask is: How many vertices can a graph have if the valence of each vertex is no greater than 3 and the distance between any two vertices is no greater than 2? Good luck on your adventure exploring this question. Is the graph below of this type?



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