

GRE MATH REVIEW #1

Basic Arithmetic

The **whole numbers**, or counting numbers, are 0,1,2,3,4,5,6... The **integers** are ...-4, -3, -2, -1,0,1,2,3,4... Recall the number line in deciding whether one negative number is larger than another. Positive integers increase as they move away from 0; negative integers decrease as they move away from 0. For example, 4 is greater than 3, but -4 is less than -3.

Consecutive integers are in increasing order without any integers missing between them. For example, 0,1,2,3,4... are consecutive integers; -2,0,2,4...are consecutive even integers; -3, -1,1,3... are consecutive odd integers.

Zero is both a whole number and an even integer, but it is neither positive nor negative. The sum of 0 and any number is that number; the product of 0 and any number is 0.

There are 10 **digits** in our number system: 0,1,2,3,4,5,6,7,8,9. An integer greater than 9 is made up of several digits. For example, in the number 5234, 4 is the ones digit, 3 is the tens digit, 2 is the hundreds digit, 5 is the thousands digit.

When multiplying **positive** and **negative** numbers, a positive times a positive is a positive; a negative times a negative is a positive; a positive times a negative is a negative:

neg + neg = neg
pos + pos = pos
neg + pos = !

(neg) x (neg) = pos
(pos) x (pos) = pos
(neg) x (pos) = neg

An **even** number is any number that can be divided evenly by 2 with no **remainder** left over. An **odd** number cannot be divided evenly by 2. Any integer is even if its ones digit is even; an integer is odd if its ones digit is odd. Don't confuse even/odd with positive/negative. On the GRE, remember the following:

even + even = even
odd + odd = even
even + odd = odd

(even) x (even) = even
(odd) x (odd) = odd
(even) x (odd) = even

An integer is **divisible** by 2 if its ones digit is divisible by 2.
An integer is **divisible** by 3 if the sum of its digits is divisible by 3.
An integer is **divisible** by 5 if its ones digit is either 0 or 5.
An integer is **divisible** by 10 if its ones digit is 0.

A **prime number** is a number that can be divided evenly by only itself and 1. 0 and 1 are not prime numbers; 2 is the only even prime number. Some examples of prime numbers are 2, 3, 5, 7, 11, 13, 17...

A number x is a **factor** of a number y if y is divisible by x . For example, 2, 3, 4, and 6 are all factors of 12 since they all divide evenly into 12.

A **multiple** of a number x is x multiplied by any integer except 0. For example, 10, 20, 30, 40, etc. are all multiples of 10.

The following is a list of **symbols** and their meanings you need to know on the GRE:

=	equal to
≠	not equal to
>	greater than
<	less than
≥	greater than or equal to
≤	less than or equal to

The following is a list of **terms** and their definitions you need to know on the GRE:

sum	the result of addition
difference	the result of subtraction
product	the result of multiplication
quotient	the result of division
numerator	the “top” number in a fraction
denominator	the “bottom” number in a fraction

The **six basic operations** you will need to perform on the GRE are as follows. Let a and b be any numbers

1. addition: $a + b$
2. subtraction: $a - b$
3. multiplication: $ab = a \bullet b = a \times b = (a)(b)$
4. division: $a/b = a \div b$
5. raising a number to an exponent: a^2
6. finding square roots and cube roots: $\sqrt{a}, \sqrt[3]{b}$

In order to find an answer when more than one operation is involved, you must know the correct order of operations. Remember the following mnemonic: Please Excuse My Dear Aunt Sally. **PEMDAS** stands for Parentheses, Exponents, Multiplication, Division, Addition, and Subtraction. This is the order in which the operations must be performed. For example, $10(2) + (11-1) \div 5 - 4 = 18$.

NOTE: MULTIPLICATION AND DIVISION GOES LEFT TO RIGHT, i.e., $10 \div 2 \times 5 = 25$ NOT 1; ADDITION AND SUBTRACTION ALSO GOES LEFT TO RIGHT, i.e., $10 - 2 + 5 = 13$ NOT 3.

The **associative** laws of addition and multiplication allow you to regroup numbers in any order when adding or multiplying. If a, b, and c are any numbers,

$$a + (b + c) = (a + b) + c \text{ and } a (b c) = (a b) c$$

For example, $2 + (4 + 6) = 4 + (2 + 6) = (6 + 4) + 2 = 2 + 4 + 6$ and $2(4 \bullet 6) = (4 \bullet 2)6 = 6(2 \bullet 4) = 6 \bullet 4 \bullet 2$.

The **distributive** laws are very important on the GRE. Apply them every chance you get. If a, b, and c are any numbers, then

$$\begin{array}{lll} a(b + c) = ab + ac & \text{and} & (b + c)a = ba + ca = ab + ac \\ a(b - c) = ab - ac & \text{and} & (b - c)a = ba - ca = ab - ac \end{array}$$

When you use the distributive law to go from $ab + ac$ to $a(b + c)$, you are actually **factoring** the expression $ab + ac$ by finding the common term to ab and ac which is “a”.

EXERCISE 1

1. List five consecutive negative even integers.
2. List five consecutive integers, one of which is 0.
3. List all the prime numbers less than 30.
4. What is the least integer greater than -5.8 ?
5. What is the greatest integer less than -3.6 ?
6. What is a 3-digit number whose digits add up to 14?
7. Without performing division, what is the remainder when:
(a) 99 is divided by 5?
(b) 12,345,671 is divided by 10?
8. A multiple of both 3 and 7 is also a multiple of what number?
9. Is the product of 34,569 and 227 odd or even?
10. Is 5223 divisible by (a) 3? (b) 2? (c) 5? (d) 10?
11. If 2 even numbers are multiplied together and then the product is multiplied by 37, will the result be even or odd?
12. Express 56 as the product of prime numbers.
13. If -3 is multiplied by -345 , is the result positive or negative? odd or even?
14. $2(4 + 10) - 6 \div 3 =$
15. $5[3(2 - 4)] \div 15 - 2 =$
16. Use the distributive law to rewrite $3(x + 2y) - 5x(y - 4)$.
17. Factor the expression $4xyz - 12xy + 2yz$.

EXERCISE 1 SOLUTIONS

1. For example, -10, -8, -6, -4, -2.
2. For example, -2, -1, 0, 1, 2.
3. 2, 3, 5, 7, 11, 13, 17, 19, 23, 29
4. -5
5. -4
6. For example, 275.
7. (a) Find the greatest multiple of 5 that is still less than 99 and find the difference between that number and 99. That multiple is 95, so the remainder is 4.
(b) The greatest multiple of 10 that is still less than 12,345,671 is 12,345,670, so the remainder is 1.
8. 21
9. odd
10. (a) yes (b) no (c) no (d) no
11. even
12. $56 = 2 \cdot 2 \cdot 2 \cdot 7$
13. positive, odd
14. 26
15. -4
16. The distributive law results in $3x + 6y - 5xy + 20x$. Although simplifying algebraic expressions has not yet been discussed, this expression can be simplified to $23x + 6y - 5xy$.
17. $2y(2xz - 6x + z)$