EXPONENTIAL AND LOGARITHMIC FUNCTIONS

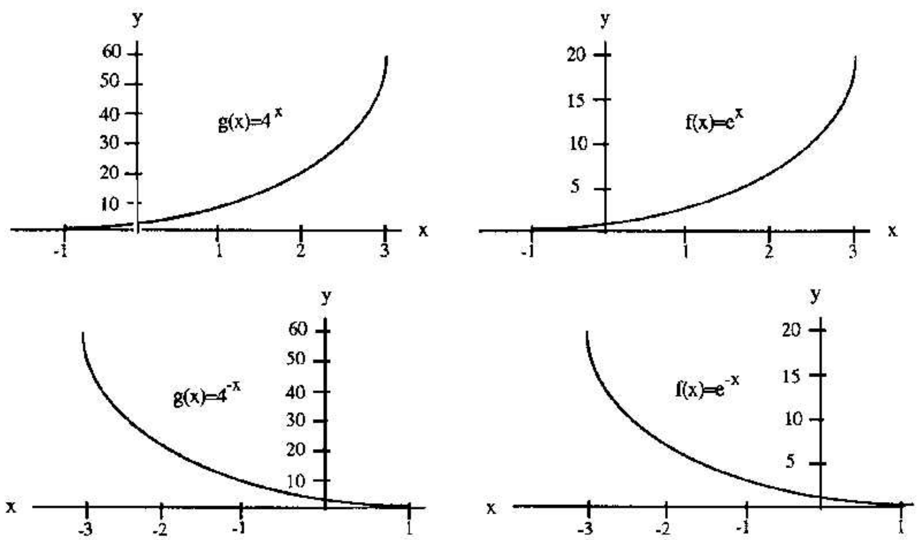
Exponential Functions:

The **exponential function** f **with base** a is denoted by

$f(x) = a^x$ where $a > 0$, $a \neq 1$, and x is any real number.

The function $f(x) = e^x$ is called the **natural exponential function.** The number $\boldsymbol{e} \approx 2.71828$ is irrational and is called the **natural base.**

Examples of Graphs of Exponential Functions:



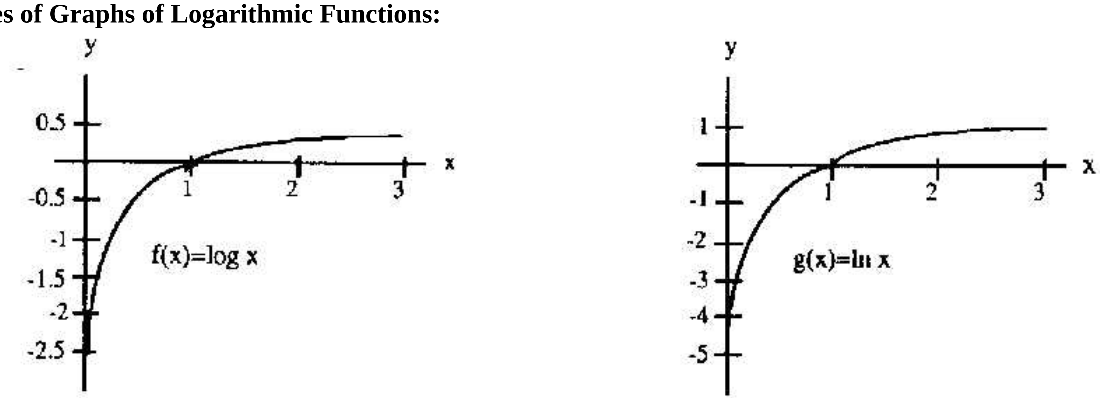
Logarithmic Functions:

The **logarithmic function with base a** is denoted by

$f(x) = \log_a x$ where $x > 0$, $0 < a \neq 1$, $y = \log_a x$ if and only if $x = a^y$

The logarithmic function with base 10 is called the **common logarithmic function** and is denoted by $f(x) = \log x$.
The function $f(x) = \log_e x = \ln x$ where $x > 0$ is called the **natural logarithmic function** and can be rewritten as ln x ($\log_e x = \ln x$).

Examples of Graphs of Logarithmic Functions:



Properties of logarithms:

$x = \log_b c$ if and only if $b^x = c$
$b^x = b^y$ if and only if $x = y$

$b^{\log_b c} = c$ note that in this case $\log_b c = x$ and thus $b^x = c$,

$\log_b b^y = y$, because $b^y = b^y$

$\log_b 1 = 0$, because $b^0 = 1$

$\log_b b = 1$, because $b^1 = b$

$\log_b (MN) = \log_b M + \log_b N$

$\log_b \left(\frac{M}{N}\right) = \log_b M - \log_b N$

$\log_b N^y = y \cdot \log_b N$

$\log_b \left(\frac{1}{N}\right) = \log_b (N)^{-1} = -1 \cdot \log_b N$

$\ln e^x = x$, because $e^x = e^x$

$\ln 1 = 0$, because $e^0 = 1$

$\ln e = 1$, because $e^1 = e$

$\ln x = \log_e x$

$\log_{10} x = \log x$

Change of Base formula:

Let a, b, and x be positive real numbers such that $a \neq 1$ and $b \neq 1$. Then

$$\log_a x = \frac{\log_b x}{\log_b a}$$

Inverse Relationship of Logarithmic and Exponential Functions:

$\ln(e^x) = x$ and $e^{\ln x} = x$
$\log(10^x) = x$ and $10^{\log x} = x$