

Math Explorations Algebra I

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MATH EXPLORATIONS

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VARIABLES, EXPRESSIONS AND EQUATIONS

1

SECTION 1.1 CONSTRUCTING A NUMBER LINE

Name: _____ Date: _____ Period: _____

Vocabulary

<u>DEFINITION</u>	<u>EXAMPLE</u>
Integers	
Natural Numbers	
Whole Numbers	
Rational Numbers	
Linear Model	

More Vocabulary

<u>DEFINITION</u>	<u>EXAMPLE</u>
Elements	
Sets	
Subset	
Set Notation	
Absolute Value	

EXPLORATION 1

Build a number line.

1. Make a number line on a large piece of paper. Put the number 1 in the middle of the line. Locate and label the first twenty natural numbers.
2. Plot and label 0 on the number line. If we include 0 and the natural numbers, what do we call the set of numbers?
3. Using a red marker, plot and label the negative integers from -1 to -20 .
4. Using a different colored marker, plot and label 3 fractions between each of the following pairs of integers:

2 and 3 4 and 5 -1 and 0 -3 and -2

EXAMPLE 1

Create a Venn Diagram to show the relationship between the following sets of numbers:

- rational numbers
- whole numbers
- integers
- natural numbers

EXPLORATION 4

1. Use the number line to illustrate the sum $3 + (-4)$ and the difference $3 - 4$. Explain how you arrived at your answer and location for each problem. Then, using the same pattern, explain how you compute the sum $38 + (-63)$ and the difference $38 - 63$ without a detailed number line.
2. Use the number line to illustrate the difference $3 - (-5)$ and sum $3 + 5$. Then explain how you compute the difference $38 - (-63)$ without a detailed number line.
3. Summarize the rules for addition and subtraction of integers.
4. Use the number line to illustrate the product $3(-4)$ and $-3(4)$. Explain how you arrived at your answer and location for each problem. Then using the same pattern, explain how you compute the products $18(-6)$ and $-5(12)$ without a detailed number line.
5. Use the number line to illustrate the product $-3(-4)$. Explain how you arrived at your answer and location for each problem. Then using the same pattern, explain how you compute the product $-28(-3)$.
6. Summarize the rules for multiplication of integers.

EXPLORATION 5

1. Use the number line to illustrate the sums $1\frac{3}{4} + 2\frac{3}{4}$ and $\frac{4}{5} + \frac{3}{5}$.

2. Starting at the point representing 3, determine and locate on the number line the following numbers. Explain how you arrived at your answer.
 - a. The number that is 5 more than this number.

 - b. The number that is 5 less than this number.

 - c. The number that is 3 times this number.

 - d. The number that is half as big as this number.

3. Locate and label three numbers that are greater than -5 . Locate and label three numbers that are less than -6 .

EXPLORATION 6

Use your number line to determine the distance between 6 and 13. How did you arrive at your answer?

1. What is the distance from 12 to 4? Explain how you got your answer.
2. What is the distance from -3 to -11 ? From -9 to -2 ? Explain how you got your answers.
3. What is the distance from -7 to 4 ? What is the distance from 5 to -7 ? Explain how you got your answers.
4. Find the distance between $\frac{1}{2}$ and $3\frac{1}{2}$.
5. Find the distance between $\frac{1}{2}$ and $\frac{3}{4}$.
6. Find the distance between $\frac{3}{4}$ and $3\frac{1}{2}$.
7. What is the distance from $-\frac{1}{2}$ to $\frac{7}{8}$?
8. What is the distance between $4\frac{2}{3}$ and $1\frac{1}{2}$?

PROBLEM 2

Compute the distance between the following pairs of numbers.

1. -12 and 6

2. -52 and 27

3. -23 and -35

4. 1.75 and -1.25

5. $\frac{3}{4}$ and $-\frac{1}{3}$

SUMMARY (What I learned today)
