MATH EXPLORATIONS PART 2

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Preface and Introduction

Math Explorations follows several fundamental principles. It is important to carefully state these at the beginning, and describe how these are a perfect fit not only in educating the general student population, but also in teaching students whose native language is not English. These guiding principles will help the curriculum come alive for all students.

Learning math is not a spectator sport. The activities that fill the text and accompanying workbooks encourage students to develop the major concepts through exploration and investigation rather than by given rules to follow. **A crucial element is to understand the importance of small-group work, and to appreciate the extent to which everyone can benefit from working together.** In fact, often the process of explaining how to work a problem helps the explainer as much or more than the person who asks the question. As every teacher knows, explaining an idea to someone else is one of the best ways to learn it for oneself.

Some basic rules for discussion within a group include:

1. Encourage everyone to participate and value each person's opinions. Listening carefully to what someone else says can help clarify a question.
2. If one person has a question, remember that the chances are someone else will have the same question. Be sure everyone understands new ideas completely and never be afraid to ask questions.
3. Don't be afraid to make a mistake. In the words of Albert Einstein, "A person who never made a mistake never discovered anything new." Group discussion is a time of exploration without criticism. In fact, many times mistakes help to identify difficulties in solving a problem. Rather than considering a mistake a problem, think of a mistake as an opportunity to learn more about the process of problem-solving.
4. Always share your ideas with one another, and make sure that everyone is able to report the group reasoning and conclusions to the class. Everyone needs to know why things work and not just the answer. If you don't understand an idea, be sure

to ask "why" it works. You need to be able to justify your answers. The best way to be sure you understand why something works is to describe your solution to the group and class. You will learn more by sharing your ideas with one another.

If an idea isn't clear there are several things to try:

1. Look for simpler cases. Looking deeply at simple cases can help you see a general pattern.
2. Ask your peers and teacher for help. Go beyond "Is this the right answer?"
3. Understand the question being asked. Understanding the question leads to mathematical progress.
4. Focus on the process of obtaining an answer, not just the answer itself; in short become problem centered, not answer centered. One major goal of this book is to develop an understanding of ideas that can solve more difficult problems as well.
5. After getting help, work the problem yourself, and make sure you really understand. Make sure you can work a similar problem by yourself.

Suggestions for responding to oral questions in group and class discussion:

As you work through the Explorations in the book, working both individually and in groups can make understanding the material easier. Sometimes it is better to explore problems together, and other times you may want to explore first by yourself and then with others by discussing your ideas. When you discuss the problems as a group, it is more productive if you try to remember these simple rules:

1. Try not to interrupt when someone else is talking.
2. In class, be recognized if you want to contribute or ask a question.
3. Be polite and listen when others in your group or class are talking. This is one of the best ways to learn.

4. Finally, if you have a question, raise your hand and ask. Remember, there is almost always someone else with a similar or identical question.

Last, some general advice about taking notes in math:

1. Reading math is a specific skill. When you read math, you need to read each word carefully. The first step is to learn the mathematical meaning of all words. Some words may be used differently in math than in everyday speech.
2. It is often necessary to write definitions of new words and to include mathematical examples. Try to write definitions in your own words without changing the meaning or omitting any important point. When you write down a definition, look for an example that illustrates what you are learning. This will help you relate what you are learning to real world situations.
3. Explaining new ideas and definitions that you read to your peers and teachers is very helpful. This will provide practice with any new definitions, and make sure that you are using the words correctly. Explaining a concept can help to correct any misconceptions and also reinforces learning.
4. If possible, try to draw a visual representation to make a difficult or new concept clear. It is really true that "a picture is worth a thousand words." Visual cues help you understand and remember definitions of new terms.

Throughout this book, students learn algebraic thinking and the precise use of mathematical language to model problems and communicate ideas. The communication that makes this possible can be in small groups, in class discussion, and in student notes.

It is important to note that the use of variables and algebra is not an afterthought, but is woven throughout all of our books. By using language purposefully in small groups, class discussions, and in written work, students develop the ability to solve progressively more challenging problems.

The authors are aware that one important member of their audience is the parent. Parents are encouraged to read both the book and the accompanying materials and talk to their students about what they are learning.

Possibly the most unique aspect of this book is the breadth and span of its appeal. The authors wrote this text for both a willing 4th or 5th grader and any 6th grader. Students may particularly enjoy the ingenuity and investigation problems at the end of each set of exercises which are designed to lead students to explore new concepts more deeply.

The text has its origins in the Texas State Honors Summer Math Camp (HSMC), a six-week residential program in mathematics for talented high school students. The HSMC began in 1990 and is modeled after the Ross program at Ohio State, teaching students to "think deeply of simple things" (A. E. Ross). Students learned mathematics by exploring problems, computing examples, making conjectures, and then justifying or proving why things worked. The HSMC has had remarkable success over the years, with over 150 students being named semi-finalists, regional finalists, and national finalists in the prestigious Siemens Competition in Math, Science, and Engineering. Initially supported by grants from the National Science Foundation and RGK Foundation, the HSMC has also received significant contributions from Siemens Foundation, Intel, SBC Foundation, Coca-Cola, the American Math Society Epsilon Fund, and an active, supportive Advisory Board.

In 1996, two San Marcos teachers, Judy Brown and Ann Perkins, suggested that we develop a pipeline to the HSMC that would introduce all young students to algebra and higher-level mathematics. Following their suggestion, we began the Junior Summer Math Camp (JSMC) as a two-week program for students in grades 4–8. We carefully developed the JSMC curriculum by meeting regularly with Judy and Ann, who gave us invaluable feedback and suggestions.

With support from the Fund for the Improvement of Postsecondary Education (FIPSE), Eisenhower Grants Program, Teacher Quality Grants, and the Texas Education Agency, we developed the JSMC into a replicable model that school districts throughout the state could implement. The JSMC curriculum was designed to prepare all students for higher-level mathematics. In some districts the JSMC targeted gifted students; in other districts the program was delivered to mixed group of students. In every setting, the program had remarkable results in preparing students for algebra as measured by the Orleans-Hanna algebra prognosis pre- and post-tests.

Over the years, we trained hundreds of teachers and thousands of students. Although we cannot thank each personally, we should mention that it has been through their suggestions and input that we have been able to continually modify, refine, and improve the curriculum.

A concern with the JSMC curriculum was that it was only supplementary material for teachers, and many of the state-required mathematics topics were not included. The Math Explorations texts that we have written have taken the JSMC curriculum and extended it to cover all of the TEKS (Texas Essential Knowledge and Skills), for grades 6-8 while weaving in algebra throughout. The third volume for 8th graders allows all students to complete Algebra I. This is an integrated approach to algebra developed especially for younger students. By learning the language of mathematics and algebra, young students can develop careful, precise mathematical models that will enable them to work multi-step problems that have been a difficult area for U. S. students on international tests.

An accompanying Teacher Edition (TE) has been written to make the textbook and its mathematical content as clear and intuitive for teachers as possible. The guide is in a three-ring binder so teachers can add or rearrange whatever they need. Every left-hand page is filled with suggestions and hints for augmenting the student text. Answers to the exercises and additional activities are also provided in the TE.

This project had wonderful supporters in the Meadows Foundation, RGK Foundation, and Kodosky Foundation. A special thanks to the Mathworks Steering Committee, especially

Bob Rutishauser and Jeff Kodosky, who have provided constant encouragement and support for our curriculum project. The person who motivated this project more than any other was Jeff Kodosky, who immediately realized the potential it had to dramatically change mathematics teaching. Jeff is truly a visionary with a sense for the important problems that we face and ideas about how to solve them. His kind words, encouragement, and support for our JSMC and this project have kept me going whenever I got discouraged.

Our writing team has been exceptional. The primary basis for the book was our JSMC curriculum, coauthored with my wife Hiroko Warshauer, and my friend and colleague, Terry McCabe. The three of us discuss every part of the book, no matter how small or insignificant it might seem. Each of us has his or her own ideas, which together I hope have made for an interesting book that will excite all young students with the joy of mathematical exploration and discovery. During the summers of 2005-2013, we have been assisted by an outstanding group of Honors Summer Math Camp alumni, undergraduate and graduate students from Texas State, as well as an absolutely incredible group of pilot teachers. While it would take a volume to list everyone, we would be remiss not to acknowledge the help and support from these past summers.

Terry McCabe led the team in developing this book, Math Explorations Part 2, assisted by Hiroko Warshauer and Max Warshauer. Alex White from Texas State provided valuable suggestions for each level of the curriculum, and took over the leadership of the effort for the third volume, Math Explorations Part 3: Algebra 1.

We made numerous refinements to the curriculum in summer 2012 incorporating the 2012 revised TEKS, additional exercises, new warm-ups, and an accompanying collection of workbook handouts that provides a guide for how to teach each section. Many of these changes were inspired by and suggested by our pilot site teachers. Amy Warshauer and Alexandra Eusebi from Kealing Middle School in Austin, who piloted Part 2 of the curriculum with their 6th grade students, helped design and put together a fabulous collection of workbook handouts that accompany the revised text. Melissa Freese from Midland provided valuable assistance on both the workbooks and in making new warm-up problems. Additional edits and proofreading was done by Michael Kellerman, who did a wonderful job making sure that the language of each section was at the appropriate grade level. Robert Perez from Brownsville developed special resources for English Language Learners, including a translation of key vocabulary into Spanish.

Math Explorations Part 1 should work for any 6th grade student, while Math Explorations Part 2 is suitable for either an advanced 6th grade student or any 7th grade student. Finally, Math Explorations Part 3: Algebra 1 completes algebra for all 8th grade students. The complete set of 3 books covers all of the Texas Essential Knowledge and Skills (TEKS) for grades 6-8 while also covering Algebra I.

The production team was led by Claudia Hernandez, an undergraduate student in mathematics at Texas State. Claudia did an outstanding job of laying out the book, editing, correcting problems, and in general making the book more user friendly for students and parents. She has a great feel for what will excite young students in mathematics, and worked tirelessly to ensure that each part of the project was done as well as we possibly could. As we prepared our books for state adoption, Bonnie Leitch came on board to help guide and support the entire project. Bonnie worked tirelessly to find where each of the Texas Essential Knowledge and Skills (TEKS) and English Language Proficiency Skills (ELPS) was covered in both the text and exercises. We added additional exercises and text to cover any TEKS that were not sufficiently addressed. Bonnie also edited these revisions and gave a final proofreading for each of the books, working with the authors to proofread every edit. However, in the end the authors take total responsibility for any errors or omissions. We do, however, welcome any suggestions that the reader might have to help make future editions better. In short, we had an incredible, hard-working team that did the work of an entire textbook company in a few short weeks! Without their help the project would not have reached its present state.

Finally, in this newest 5th edition (2016), we have continued to make edits and improvements. Claudia Hernandez and Sammi Yarto, undergraduates at Texas State, did a fabulous job of adding in these corrections, formatting the text to be consistent, and correcting any typos. Everyone at Mathworks contributed to the final product, especially Michelle Pruett, our new curriculum coordinator, and Patty Amende, who helped oversee the entire project. We could never have completed the revisions without the incredible help and dedication of our whole team.

Any curriculum will only be as effective as the teachers who use it, and without the support and encouragement of the administration and parents, this can never happen. In this, we have been very fortunate to be able to work with dedicated teachers and administrators from San Marcos, McAllen, New Braunfels, Midland, and Austin. The Mathworks staff gave invaluable help. Michelle Pruett, Andrew Hsiau, and Patty Amende provided support whenever needed. I hope you will join our team by giving us feedback about what works, what doesn't, and how we can improve the book. By working together, I believe that we can develop a mathematics curriculum that will reach out to all students and that will engage students at a higher level than we have previously been able to achieve.

Max Warshauer
Director of Texas State Mathworks

MATH EXPLORATIONS

Preface and Introduction

SECTION 1.1
BUILDING NUMBER LINES

BIG IDEA

Constructing number lines and modeling integers.

KEY OBJECTIVES

- Use the linear model to order numbers: number lines.
- Realize importance of order and spacing when constructing number lines.
- Identify types of numbers (counting, natural, whole, and integers).
- Compare and order integers within the context of temperature.

PEDAGOGICAL/ORCHESTRATION

- Several of the homework questions require students to think about and draw a thermometer. You might want to have one handy to show or pass around. The big ones for outside patios and gardens would be easy to read.
- Within launch, include background of Roman Numbers being adopted from letters—does not have a zero, starts with counting numbers.

EXERCISES OF NOTE

Exercise 13 (Ingenuity) foreshadows operations with integers

Exercise 14 (Investigation) is a good problem for a discussion on proper scaling of number lines

MATERIALS

- Thermometer
- Objects to count (bananas might be fun, but counters are more useful)
- Colored Tape
- Index cards

ACTIVITY

"United We Stand"

VOCABULARY

set model, number line model, origin, counting numbers, positive integers, natural numbers, whole numbers, non-negative integers, integers, negative integers, trichotomy, degrees, Celsius, Fahrenheit

TEKS

6.1C, 6.2A, 6.2C

(1) Mathematical process standards. The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:

- (A) apply mathematics to problems arising in everyday life, society, and the workplace.

(2) Number and operations. The student applies mathematical process standards to represent and use rational numbers in a variety of forms. The student is expected to:

- (A) classify whole numbers, integers, and rational numbers using a visual representation such as a Venn diagram to describe relationships between sets of numbers;
- (C) locate, compare, and order integers and rational numbers using a number line.

7.1C, 7.2

(1) Mathematical process standards. The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:

- (C) select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate, and techniques, including mental math, estimation, and number sense as appropriate, to solve problems;

(2) Number and operations. The student applies mathematical process standards to represent and use rational numbers in a variety of forms. The student is expected to extend previous knowledge of sets and subsets using a visual representation to describe relationships between sets of rational numbers.

8.1C, 8.2A

(1) Mathematical process standards. The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:

- (C) select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate, and techniques, including mental math, estimation, and number sense as appropriate, to solve problems.

(2) Number and operations. The student applies mathematical process standards to represent and use real numbers in a variety of forms. The student is expected to:

- (A) extend previous knowledge of sets and subsets using a visual representation to describe relationships between sets of real numbers.

WARM-UPS FOR SECTION 1.1

1. On cold days in some areas, the temperature can be negative, or less than 0. Can you think of other examples in which negative numbers are used? Can you find any examples where negative numbers are better than positive ones?

   Possible answers: Negative yards for the team you are not cheering for, negative debt

2. Max was sitting in his math class and his mind began to wander. His teacher, Ms. Polly Hedron, was talking about number lines, and Max began thinking of places outside of his classroom where he might see examples of number lines. List as many examples as you can to help Max out and explain why each example reminds you of a number line.

   Possible Answers: Street addresses, thermometers, football fields, classroom numbers in hall way, clocks, rulers, etc.

LAUNCH FOR SECTION 1.1

Materials: Counters or other objects (such as bananas)

- Lead the class through the beginning of the section by modeling the idea of the set model using bananas or other counters.
- Ask the class, "Why is this called the 'set model'?" A possible response might be that numbers are thought of as sets of objects (e.g. bananas). Another possibility is that a number can describe how many objects are in a set, or a collection, of objects.
- Ask students, "In the set model, what does adding objects to a collection do to the total set?" Students will respond that it makes the set larger. "What about subtraction?" The response should be that it makes the total set smaller. The set model mostly describes "How many?" as in, "How many bananas do I have?"
- Tell the class that there are other models for numbers, and we are going to learn one that answers "How far and where?"
- Ask the students if they can think of examples in the real world where people use negative numbers. There are some examples below:
  - For a sports example, use yards gained or lost in attempting a first down.
  - The old, but good, standby of very cold temperatures and warm temperatures.
- Tell students they will be expanding their knowledge of number lines in today's lesson.

ALTERNATIVE TO EXPLORATION: CONSTRUCTING A NUMBER LINE

Materials: Colored tape

- In this class activity, you might want the students to draw their line on the board, or use tape to draw the line on the floor.
- Some important issues to discuss include:
  - Are the numbers evenly spaced?
  - How big is big? (i.e. where are the large numbers located?)
  - How do you decide which numbers are greater and which numbers are less?
  - Does the origin have to be placed in the center of the line?
- For an alternative to this exploration, see the activity "United We Stand" at the end of this section.

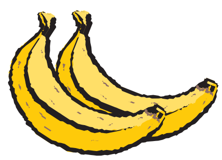
EXPLORING INTEGERS ON THE NUMBER LINE

1

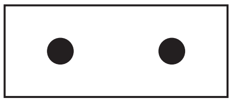
SECTION 1.1
BUILDING NUMBER LINES

Let's begin by thinking carefully about numbers. Numbers are part of the mathematical alphabet, just like letters are used in English to form words. We use numbers for counting and representing quantities. When we think of the number one, we have in mind a picture:

Similarly, the number two describes a different quantity:



We could use a picture with dots to describe the number two. For instance, we could draw:



We call this way of thinking of numbers the "**set model**." There are, however, other ways of representing numbers.

Another way to represent numbers is to describe locations with the **number line model**, which is visually similar to a thermometer. To construct a number line, begin by drawing a straight line and picking some point on the line. We call this point the **origin**. Label the origin with the number 0. We can think of 0 as the address of a certain location on

(1)

Introduce the number line to students. It is likely many will have already used one before, but there might be aspects of the number line they don't know well or don't fully understand. For example, point out that although we may draw only part of a number line, it does continue past what we can draw. It "goes on forever," and we use arrows to indicate this. As you draw a number line, have the students draw one as well in their notebooks. Go through the vocabulary (counting numbers, whole numbers, integers, etc.) as you add to your number line.

Point out that zero is very important. It is the origin, the starting place, and we use it as a reference point to describe positive and negative numbers; those to the right of zero are positive and those to the left of zero are negative. Here we see how the number line helps us think of numbers as locations. Where you are on the number line relative to zero is important.

You might consider having students either highlight vocabulary on their own or write their own glossary. Suggest using index cards that are hole-punched (to be placed in a 3-ring binder) to write new words and their meaning.

As they mark off points, check to see if they use equal spacing. You can make reference to a ruler. This foreshadows the idea of distance. You can check for spacing by using string. For example, measure distance from 0 to 3 and compare it to distance from 2 to 5. Are they the same?

Throughout this book your students will be exposed to different forms of mathematical notation. The "..." known as an ellipsis, in math means that the series continues in the same way it starts. For instance, in Definition 1.1, students can assume that the number after "7" is "8" in the first sequence and that the number after "$^{+}7$" is "$^{+}8$" in the second sequence. Another inference with the ellipsis is that the sequence continues without end.

Option: You can use the activity "United We Stand" on page 20 in having students construct a number line with both positive and negative integers at the same time.

the number line. Notice that the line continues in both directions without ending. We show this with arrows at the ends of the line.

Next, mark off some distance to the right of the origin and label the second point with the number 1.

Continue marking off points the same distance apart as above and label these points with the numbers 2, 3, 4, and so on.

The points you have constructed on your number line lead us to our first definition.

DEFINITION 1.1: COUNTING NUMBERS (POSITIVE INTEGERS)
The **counting numbers** are the numbers in the following never-ending sequence: 1, 2, 3, 4, 5, 6, 7, …   We can also write this as $^+1, ^+2, ^+3, ^+4, ^+5, ^+6, ^+7, \ldots$ These numbers are also called the **positive integers** or **natural numbers**.

One interesting property of the natural numbers is that there are "infinitely many" of them; that is, if we write down a list of natural numbers, there is always some natural number that is not on the list.

When we include the number 0, we have a different collection of numbers that we call the **whole numbers**.

(2)

Emphasize regularly that left of zero is negative and right of zero is positive on the number line. Right now this directionality only involves identifying integers, but it is also essential in computation with integers.

What is the relationship between the set of integers and the sets of whole numbers and natural (counting) numbers?

When discussing trichotomy, you might discuss it in contrast to parity (evenness or oddness). An integer can be either even or odd (2 possibilities). Ask them what 0 is and why. Some acceptable explanations include:

1. 0 is a number that is divisible by 2, and any number divisible by 2 is even.
2. Every other number is either even or odd, and if 1 is odd, then 0 must be even.

Then say that when it comes to position on the number line, there are three possibilities:

1. positive, to the right of zero;
2. negative, to the left of zero; and
3. neither positive nor negative, but exactly zero.

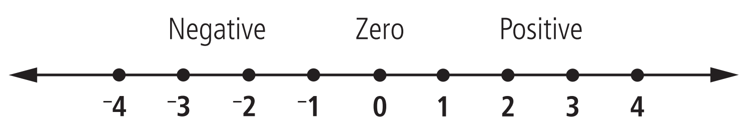
Don't worry about trichotomy too much; it's mostly a useful word for students who are confused about the difference between positive, negative, and zero.

DEFINITION 1.2: WHOLE NUMBERS (NON-NEGATIVE INTEGERS)
The **whole numbers** are the numbers in the following never-ending sequence: $0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, \ldots$ These numbers are also called the **non-negative integers.**

In order to label points to the left of the origin, we use **negative integers**:

$$^{-}1, ^{-}2, ^{-}3, ^{-}4, \ldots$$

The sign in front of the number tells us on which side of zero the number is located. Positive numbers are to the right of zero; negative numbers are to the left of zero. *Zero is not considered to be either positive or negative.*



So we have seen that numbers can be used in different ways. They can help us to describe the quantity of objects using the set model or to denote a location using the number line model. Notice that the number representing a location also can tell us the distance the number is from the origin if we ignore the sign.

DEFINITION 1.3: INTEGERS
The collection of **integers** is composed of the negative integers, zero, and the positive integers: $\ldots, ^{-}4, ^{-}3, ^{-}2, ^{-}1, 0, 1, 2, 3, 4, \ldots$

Definition 1.3 leads to the **trichotomy** property, which states that there are exactly three possibilities for an integer: positive, negative, or zero.

Such collections of numbers are often called sets of numbers. For example, assume ***S*** is the set {1, 2, 3, 4, 5} and ***T*** is the set {2, 3, 4}. Notice that every number in ***T*** is also an **element**, or member, of the set ***S***. That means that ***T*** is a **subset** of ***S***. This relationship

Reflect on the difference between representing the positive integers and negative integers using the Venn diagrams and then using the number line to represent these two sets of numbers.

What does the rectangle represent?

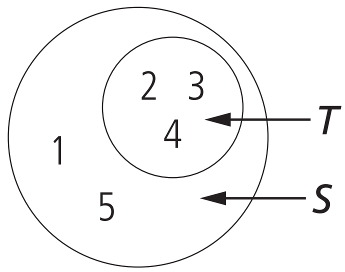
The set of real numbers.

EXPLORATION: CONSTRUCTING A NUMBER LINE

This activity is for students to practice making a careful number line. For this activity, have the students turn their papers horizontally. Parts 1, 2, and 3 require that the students draw and label $^{-}10$ and 10 on their papers. Some students may go all the way to the edge of their papers. As you get to part 4, the students may observe that they needed more room in order to plot the larger numbers. This is okay, and the student would not need to redraw the number line since part 4 only requires you to IDENTIFY approximately where these larger numbers belong. For example: at the end of the desk, in the next classroom, outside the building, in the next room, etc. This can bring out a good discussion foreshadowing how you decide what numbers to use on your number line (intervals and spacing).

4. All to the right. $^{+}10$ further from 10, $^{+}10$ further from 20, $^{+}20$ further from 30, and so on.

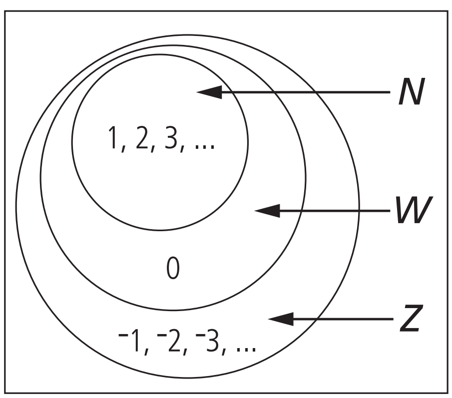
is represented by the following Venn Diagram. A Venn Diagram uses overlapping circles to organize data.



EXAMPLE 1

Call the set of positive integers N, the set of whole numbers W, and the set of integers Z. Draw a diagram of the relationship of these three sets.

SOLUTION



N= {1, 2, 3, ...}
W= {0, 1, 2, ...}
Z= {..., -3, -2, -1, 0, 1, 2, ...}

EXPLORATION: CONSTRUCTING A NUMBER LINE

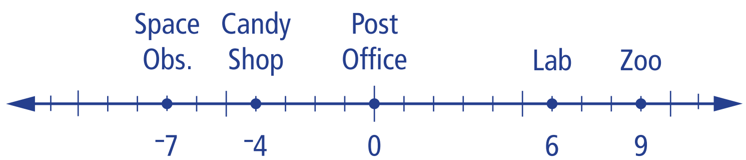
1. Draw a straight line.
2. Pick a point on the line and call this point the origin. Label the origin with the number 0.
3. Locate the numbers 1, 2, 3, …, 10, and $^-1$, $^-2$, $^-3$, …, $^-10$.
4. Where would 20, 30, 50 be located? 100? 1000?

(4)

EXERCISES

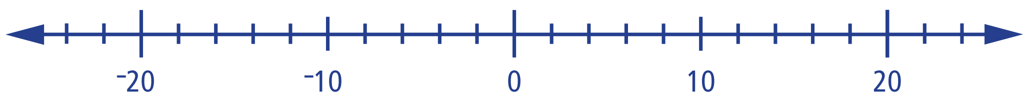
Teacher Tip: Have students create a number line from about -20 to 20 on a half sentence strip and punch holes in it so they can keep it in their binders. Students can have a horizontal number line and a vertical number line. They can also write "clue" words that identify negative direction and positive direction. You may suggest using a "folding" technique to ensure that the number line is evenly spaced.

1.



2. a. Be careful labeling. You might not have enough room. Students should be able to label every number, so the student number line should be larger than the example below.

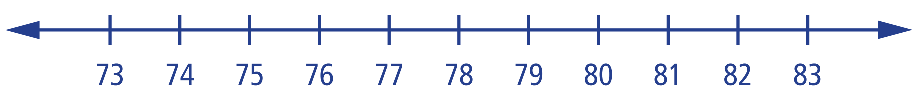
b.



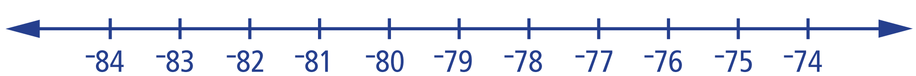
For Exercises 3–4,

- Ask students, "Why won't the origin 0 be on these number lines?"
- Be sure to remind students that 77 and -77 do not need to be in the middle of their number line.

3.



4.



5. a. 50, 50. Yes.

b. 10, 10, 10. Yes. They should check their spacing between these points in the number line.

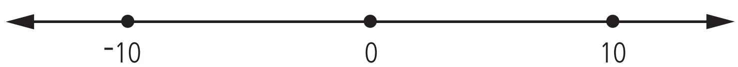
c. Answers may vary.

5. Find the negative numbers corresponding to the numbers in question 4.

EXERCISES

1. The post office is located at the origin of Main Street. We label its address as 0. The laboratory has address 6 and the zoo has address 9. Going in the other direction from the origin, we find a candy shop with address ⁻4 and a space observatory with address ⁻7. Draw a number line representing Main Street. Label each of the above locations on the number line. Watch your spacing.

2. a. Copy the line below to mark off and label the integers from 0 to 10 and from 0 to ⁻10. Use a pencil to experiment because you might need to erase. Watch your spacing.

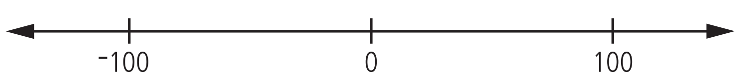


   b. Make a number line from ⁻20 to 20.

3. Draw a section of the number line containing the number 77. Mark the number 77 on your line. Now label your number line with the first few integers to the right of 77 and the next few to the left of 77, at least three each way.

4. Do the same thing you did in the previous exercise, but this time start with the number ⁻77 instead of 77.

5. Use a line like the one below to mark off the numbers with equal distances by tens from 0 to 100 and from 0 to ⁻100. Use a pencil to experiment.



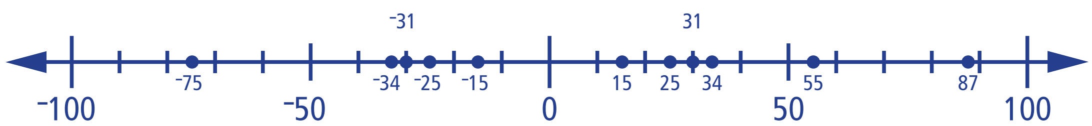
   a. What is the distance from 0 to 50 and from 50 to 100? Are they the same?

   b. What are the distances from 10 to 20, 30 to 40, and 70 to 80? Are they the same?

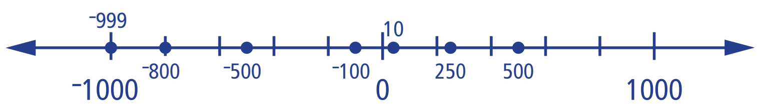
   c. Explain whether you need to rework your markings on the number line.

(5)

5. d.



6. Plotting a point is an exact placement whereas sketching is an estimation.



7.

8. The number to the right is greater.

   This problem foreshadows the next section and will give students practice with creating their own mathematical language.

9. Jan, Dec, Feb, Nov, Mar, Oct, Apr, Sept, May, Aug, Jun, Jul

   Cold ———————————▶ Warm

10. -7 °C is closer by 2 °C.

d. *Estimate* the locations of the following numbers and label each on your number line:

15, 25, 55, ⁻15, ⁻25, ⁻75, 34, 31, ⁻34, ⁻31, 87

6. Draw a number line so that the number ⁻1000 is at the left end and 1000 is at the right end. *Estimate* the locations of the following integers:

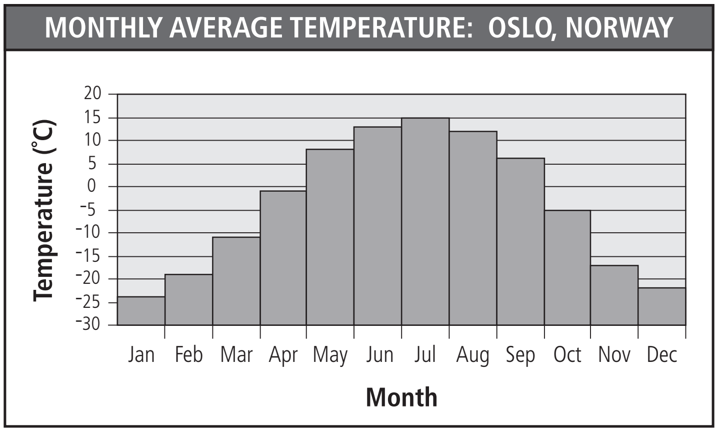
500, ⁻500, 250, ⁻100, ⁻800, 10, ⁻990, 342, ⁻781, 203, ⁻407

7. Draw a number line. Find all the integers on your number line that are greater than 15 and less than 20. Color each of these locations blue.

8. Given two numbers on the number line, how do you decide which number is greater?

Notice that we can move the number line from the horizontal position to a vertical position. We would then have a number line that looks like a thermometer. Draw a thermometer (vertical number line) on the side of your paper to help you answer questions 9 through 12.

9. The chart below shows the monthly average temperatures for the city of Oslo, Norway. Based on the data, put the twelve months in order from coldest to warmest.

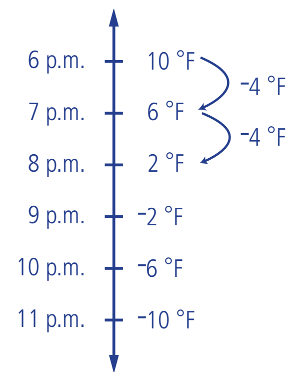


10. Chris visits Edmonton, Canada, where it is ⁻7 °C. Carmen visits Winnipeg, Canada, where it is 9 °C. Which temperature is closer to the freezing point? Explain. Remember, when we measure temperature in degrees Celsius (°C), 0 °C is the

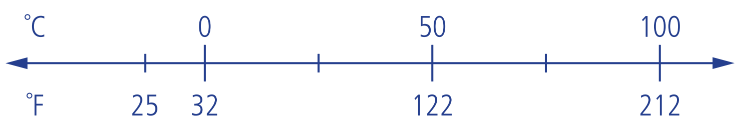
(6)

11. 4 °C is closer by 6 °C.
12. $^{-}20$ °F is colder by 5 °F.
13. **Ingenuity:**

    $^{-}10$ °F. Some students may find the following graphic useful.



14. **Investigation:**



a. 100 °C, 212 °F

b. 0 °C, 32 °F

c. 122 °F

d. Negative

e. 25 °C

f. 38 °C

Students may go to outside resources or their science class to determine the freezing and boiling points of water. Students can describe and discuss the distance between the two benchmark points in each of the two scales. The students can then see how the distance in Celsius is 100 degrees while the distance in Fahrenheit is 180 degrees. Other benchmark temperatures such as 50 and 25 degrees Celsius are then easier to find.

SUMMARY

Ask the students to describe how the number line model differs from the set model.

Answer: Direction and location are important; can visually see negative numbers.

Ask them for examples of number lines in daily life.

Answer: Thermometers, streets with addresses, rulers, classroom door numbers in hallways. For these examples ask which ones include a 0 point and where it is located.

- Streets might not include a 0. Although streets often have an intersection representing 0, they do not necessarily have a building at 0. In these cases, the 0 intersection often marks the difference between addresses known as 119 South Guadalupe vs. 119 North Guadalupe OR 13 West Congress vs. 13 East Congress. Ask the class whether North or South would correspond to negative or positive? East or West?
- Rulers are number lines with a zero, sometimes unmarked, but always the left edge of the ruler. Ask why rulers don't have negatives.

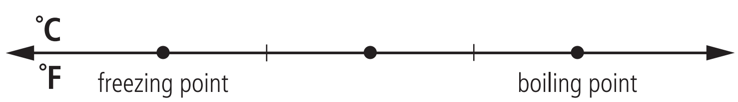
freezing point of water.

11. The temperature in Toronto, Canada, one cold day, is $^{-}10$ °C. The next day the temperature is 4 °C. Which temperature is closer to the freezing point?

12. The temperature in Fargo, North Dakota, is $^{-}15$ °F while it is $^{-}20$ °F in St. Paul, Minnesota. Which temperature is colder? How much colder?

13. **Ingenuity:**
    On a cold winter day in Iowa, the temperature at 6:00 p.m. is 10 °F. If the temperature decreases an average of 4 °F for each of the next five hours, what will the temperature be at 11:00 p.m.?

14. **Investigation:**
    Use the number line below as a thermometer with the Celsius scale above the line and the Fahrenheit scale below the line to discover how the two scales are related. Write the temperatures above and below the number line.



a. At what temperature does water boil on each scale?

b. At what temperature does water freeze on each scale?

c. What is the Fahrenheit reading for 50 °C?

d. Is the Celsius reading for 25 °F a positive or negative number?

e. A nice day is 77 °F. What is this temperature in Celsius?

f. A hot day is 100 °F. Estimate this temperature in Celsius.

UNITED WE STAND

DESCRIPTION

The students will work collaboratively to physically position themselves on a number line.

OBJECTIVE

The students will be able to explain the importance of scaling and will be able to order integers on a number line.

SEQUENCING

This activity is designed to be used as guided discovery at the beginning of Section 1.1.

MATERIALS

- Index Cards, numbered with random integers. You will need one card per student. (Be sure to include positives, negatives, and zero when you are numbering your cards.) You may wish to make two or more sets of cards, one with integers from $^{-}20$ to 20, and one with integers from $^{-}100$ to 100.
- Colored Tape

ACTIVITY INSTRUCTIONS

- Before the students get to class, use the colored tape to make an unmarked number line along the floor in the front of your room. Do not mark intervals on the number line.
- After shuffling the cards to make sure they are in random order, pass one card to each student.
- Once all of the cards have been handed out, the students will come up one at a time and position themselves on the number line that you made on the floor. Allow the important issue of spacing to arise naturally as the students arrange themselves.
- When all students are standing in the correct order at the front of the room, take this opportunity to ask them some extension questions to check for understanding of the lesson. Some examples of questions you could ask are:
  - Who is standing in the middle of the line, and why?
  - If we had a card with the number one million, where would it go?
  - If we had a card with the number _______, it would be between which two students?
  - Who represents the biggest number on our line?
  - Who represents the smallest number on our line?
  - Is the spacing between integers important?

Optional: Do the activity again, using larger numbers so that students must use a different scale.