Compound Interest

1. Simple Interest: Interest that is computed on the original principle only. \( I \) denotes the interest on a principle \( P \) (in dollars) at an interest rate of \( r \) per year for \( t \) years.

\[
I = Prt
\]

2. Accumulated Amount: is the sum of the principle and interest after \( t \) years, denoted by \( A \)

\[
A = P(1 + rt)
\]

3. Compound Interest: interest that is periodically added to the principle and then itself earns interest at the same rate.

**Compound Interest Formula (Accumulated Amount)**

\[
A = P(1 + i)^n
\]

where \( i = \frac{r}{m} \), \( n = mt \), and

- \( A \) = Accumulated amount at the end of \( n \) conversion periods
- \( P \) = Principle
- \( r \) = Nominal interest rate per year
- \( m \) = Number of conversion periods per year
- \( t \) = Term (number of years)

**Continuous Compound Interest Formula**

\[
A = Pe^{rt}
\]

where

- \( P \) = Principle
- \( r \) = Nominal interest rate compounded continuously
- \( t \) = Time in years
- \( A \) = Accumulated amount at the end of \( t \) years

4. Conversion Period: The interval of time between successive interest calculations
**Effective Rate of Interest Formula**

The annual rate of interest that, when compounded annually, will yield the same accumulated amount as the nominal interest rate compounded $m$ times a year (over the same term).

$$ r_{eff} = \left(1 + \frac{r}{m}\right)^m - 1 $$

where

$r_{eff} =$ Effective rate of interest  
$r =$ Nominal interest rate per year  
$m =$ Number of conversion periods per year

5. Present Value: the principle $P$ is often referred to as the present value

**Present Value Formula for Compound Interest**

$$ P = A(1 + i)^{-n} $$

**Present Value Formula for Continuous Compound Interest**

$$ P = Ae^{-rt} $$

6. Future Value: the accumulated amount $A$ is often referred to as the future value
Annuities

1. Ordinary Annuity: An annuity in which the payments are made at the end of each payment period (annuities where payments are made at the beginning are called an annuity due)

2. Simple Annuity: An annuity in which the payment period coincides with the interest conversion period (if it differs, it is called a complex annuity)

**Future Value of an Annuity**
The future value $S$ of an annuity of $n$ payments of $R$ dollars each, paid at the end of each investment period into an account that earns interest at the rate of $i$ per period, is

$$S = R \left[ \frac{(1 + i)^n - 1}{i} \right]$$

**Present Value of an Annuity**
The present value $P$ of an annuity consisting of $n$ payments of $R$ dollars each, paid at the end of each investment period into an account that earns interest at the rate of $i$ per period, is

$$P = R \left[ \frac{1 - (1 + i)^{-n}}{i} \right]$$
Amortization

**Amortization Formula (Periodic Payment)**

The periodic payment $R$ on a loan of $P$ dollars to be amortized over $n$ periods with the interest charged at the rate of $i$ per period is

$$R = \frac{Pi}{1 - (1 + i)^{-1}}$$

**Amortization Formula (Amount Amortized)**

By thinking of the monthly loan repayments $R$ as the payments in an annuity, we see that the original amount of the loan is given by $P$, the present value of the annuity.

$$P = R \left[ \frac{1 - (1 - i)^{-n}}{i} \right]$$

1. **Sinking Fund**: an account that is set up for a specific purpose at some future date

**Sinking Fund Payment**

The period payment $R$ required to accumulate a sum of $S$ dollars over $n$ periods with interest charged at the rate of $i$ per period is

$$R = \frac{iS}{(1 + i)^n - 1}$$
Applications of the Definite Integral to Business and Economics

1. Consumers’ Surplus: The difference between what the consumers would be willing to pay for $\bar{x}$ units of the commodity and what they actually pay for them

**Consumers’ Surplus**

The consumers’ surplus is given by

$$CS = \int_0^{\bar{x}} D(x)dx - \bar{p}\bar{x}$$

where $D(x)$ is the demand function, $\bar{p}$ is the unit market price, and $\bar{x}$ is the quantity sold. The equation can also be written as

$$CS = \int_0^{\bar{x}} [D(x) - \bar{p}]dx$$

2. Producers’ Surplus: The difference between what the suppliers actually receive and what they would be willing to receive

**Producers’ Surplus**

The producers’ surplus is given by

$$PS = \bar{p}\bar{x} - \int_0^{\bar{x}} S(x)dx$$

where $S(x)$ is the supply function, $\bar{p}$ is the unit market price, and $\bar{x}$ is the quantity supplied. The equation can also be written as

$$PS = \int_0^{\bar{x}} [\bar{p} - S(x)]dx$$

3. Accumulated (Total) Future and Present Value of an Income Stream

The accumulated, or total, future value after $T$ years of an income stream of $R(t)$ dollars per year, earning interest at the rate of $r$ per year compounded continuously, is given by

$$A = e^{rT} \int_0^T R(t)e^{-rt}dt$$

The present value of an income stream of $R(t)$ dollars per year, earning interest at the rate of $r$ per year compounded continuously, is given by

$$PV = \int_0^T R(t)e^{-rt}dt$$
3. Amount of an Annuity: the sum of payments plus the interest earned

**Amount of Annuity**
The amount of an annuity is

\[ A = \frac{mP}{r} (e^{rT} - 1) \]

where

- \( P \) = Size of each payment in the annuity
- \( r \) = Interest rate compounded continuously
- \( T \) = Term of the annuity (in years)
- \( m \) = Number of payments per year

**Present Value of an Annuity**
The present value of an annuity is given by

\[ PV = \frac{mP}{r} (1 - e^{-rT}) \]

where \( P, r, T, \) and \( m \) are as defined earlier.

4. Lorenz Curve: a method used by economists to study the distribution of income in a society

**Coefficient of Inequality of a Lorenz Curve**
The coefficient of inequality, or Gini Index, of a Lorenz curve is

\[ L = 2 \int_0^1 [x - f(x)]dx \]